Reactive Synthesis

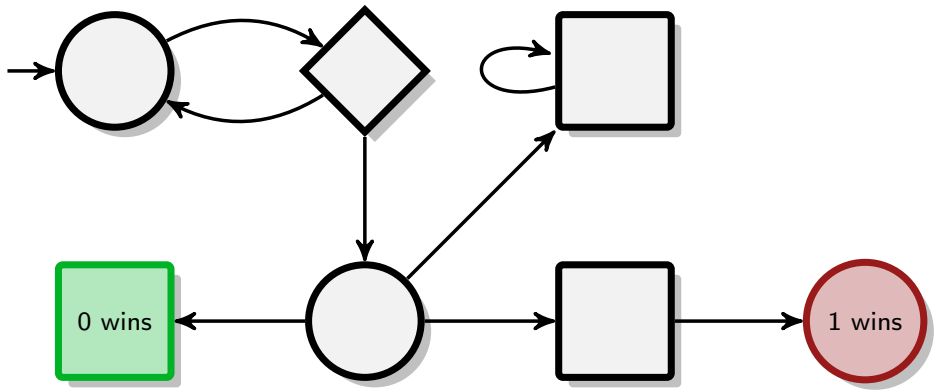
Lecture 12

Swen Jacobs and Martin Zimmermann
(Saarland University)

Outlook

(Simple) Stochastic Games

Enter a new player Nature (◇): it flips a coin to pick a successor.



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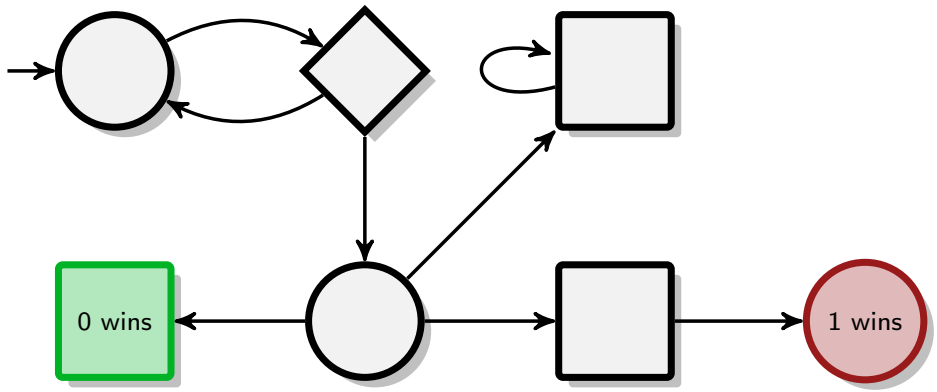

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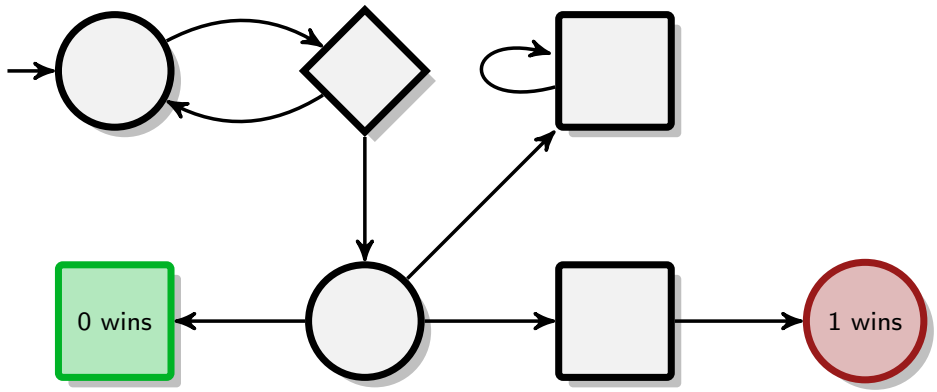

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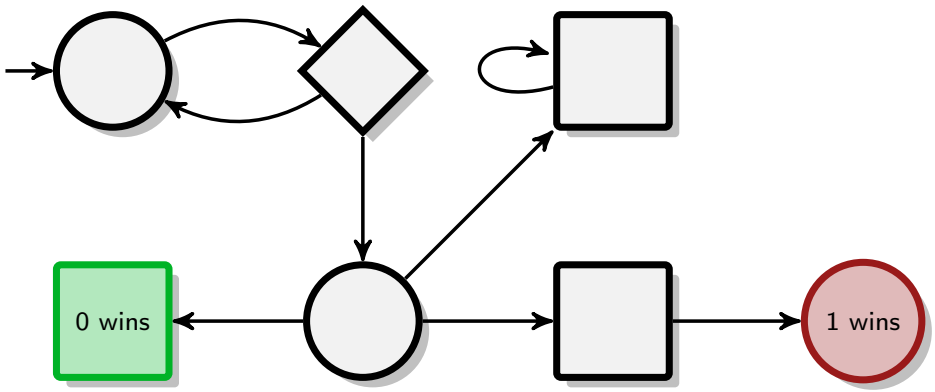


- No (sure) winning strategy... but one with probability 1.
- Value of the game for Player 0:

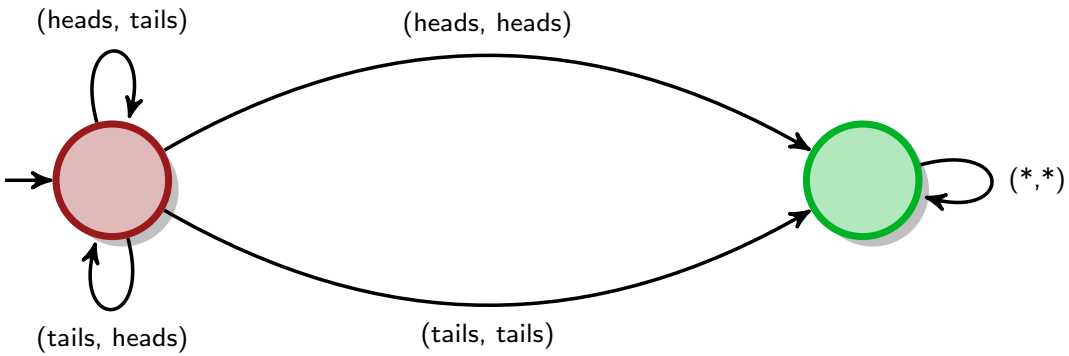
$$\max_{\sigma} \min_{\tau} p_{\sigma,\tau}$$

where $p_{\sigma,\tau}$ is the probability that Player 0 wins when using strategy σ and Player 1 uses strategy τ.

Concurrent Games

Both players choose their moves simultaneously.

- Matching pennies:



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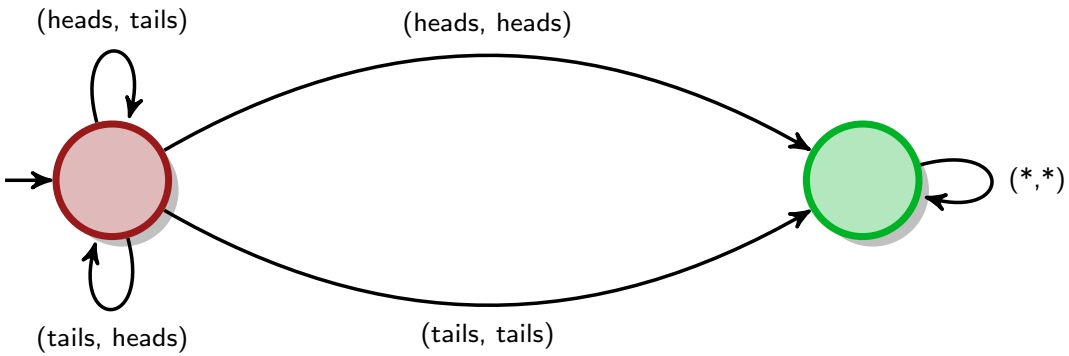

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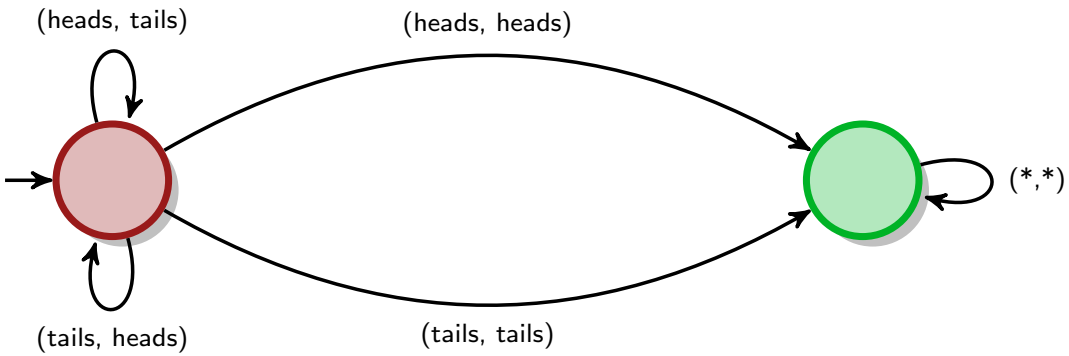
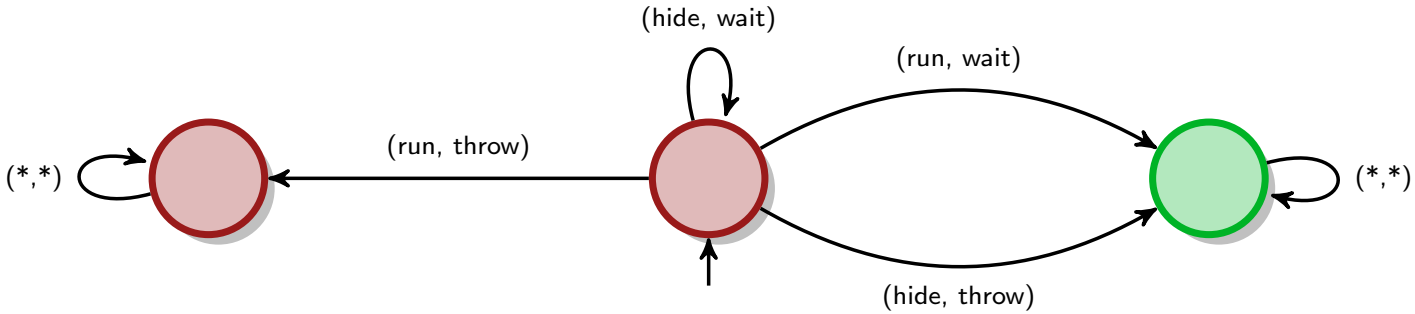


- The "Snowball Game":



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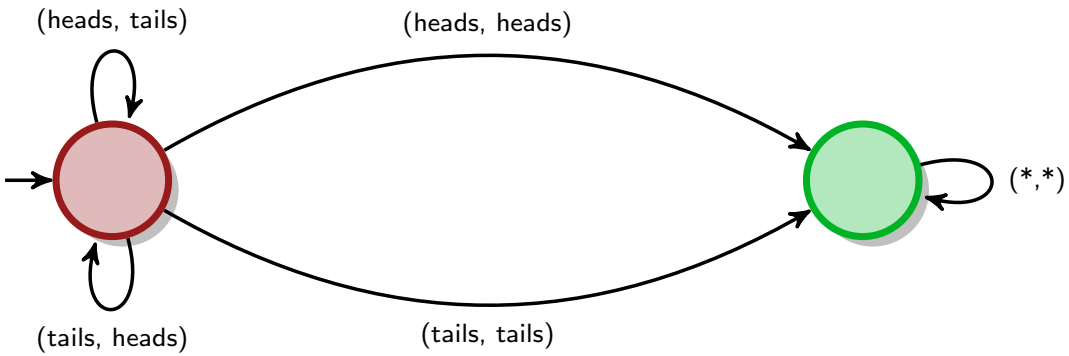
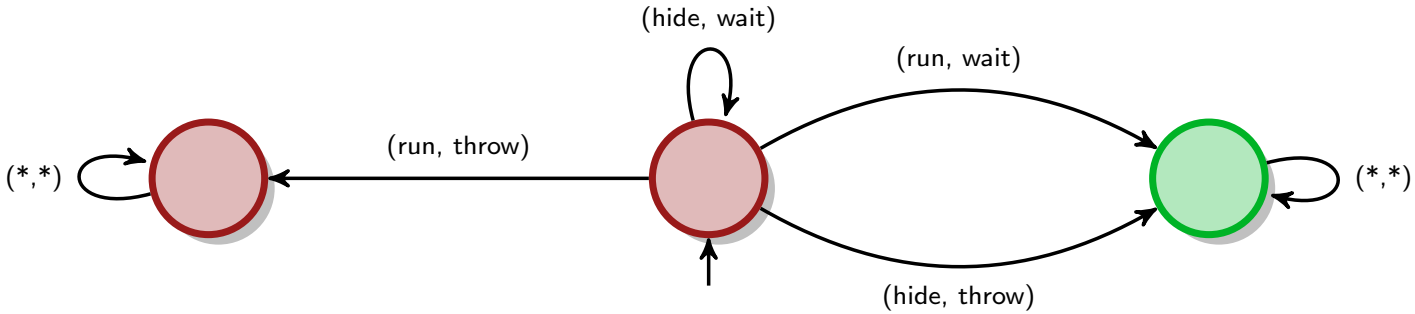
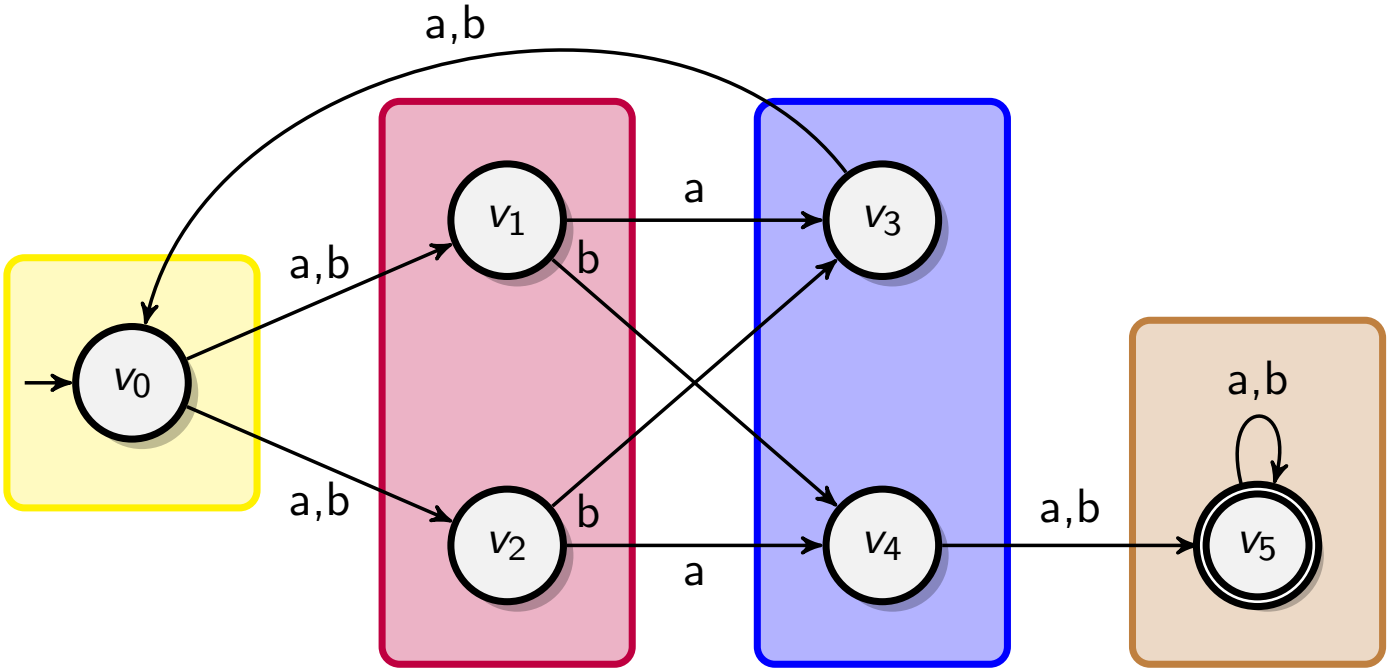


- The "Snowball Game": for every ε, randomized strategy winning with probability $1 - \varepsilon$.



Games of Imperfect Information

- Players do not observe sequence of states, but sequence of non-unique observations (yellow, purple, blue, brown).
- Player 0 picks action a/b, Player 1 resolves non-determinism.



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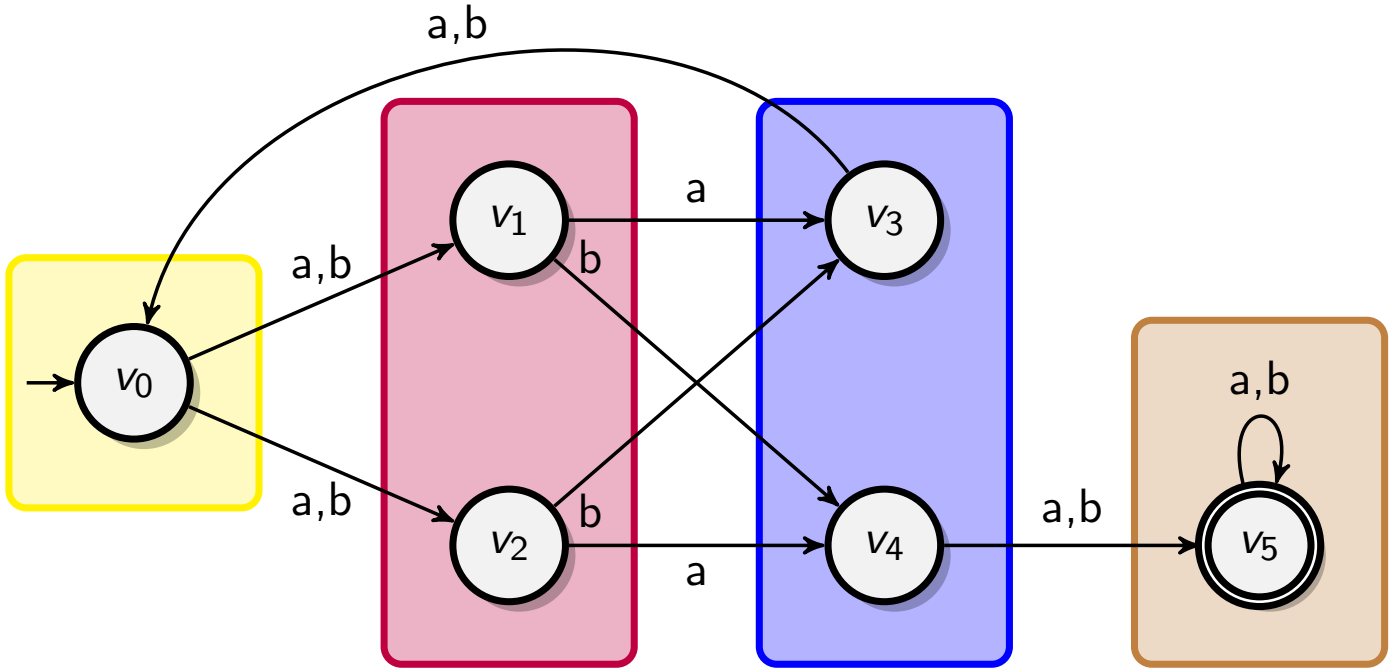
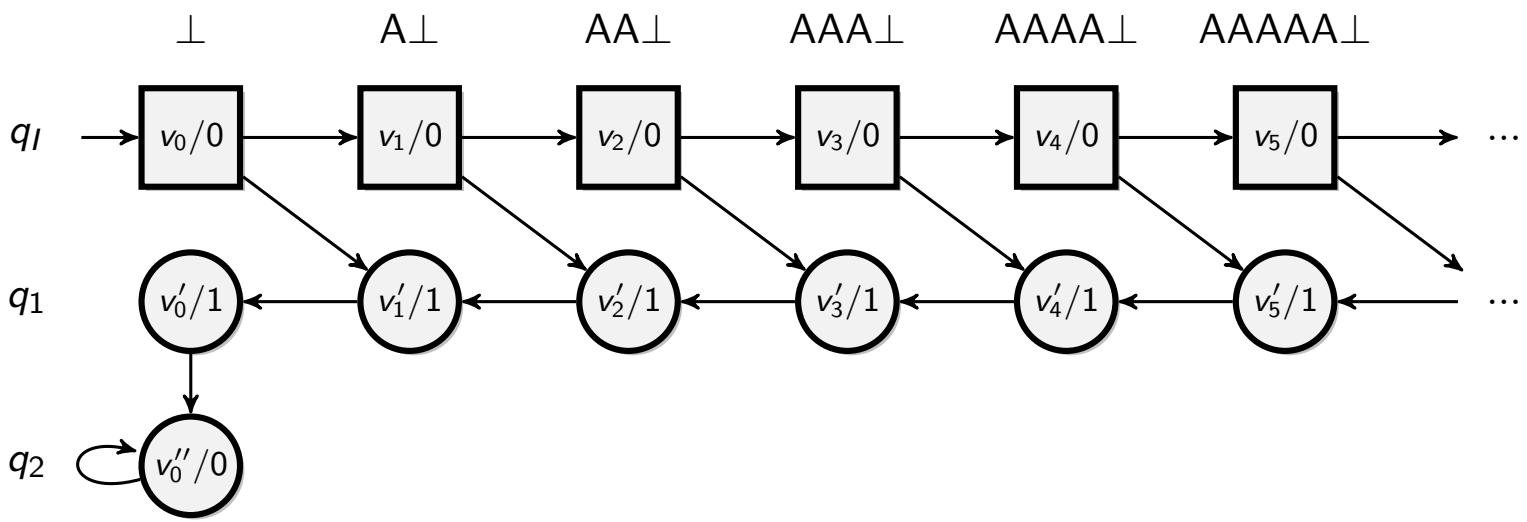


No winning strategy for Player 0: every fixed choice of actions to pick at ( )*( ) can be countered by going to v_1 or v_2.

Pushdown Games



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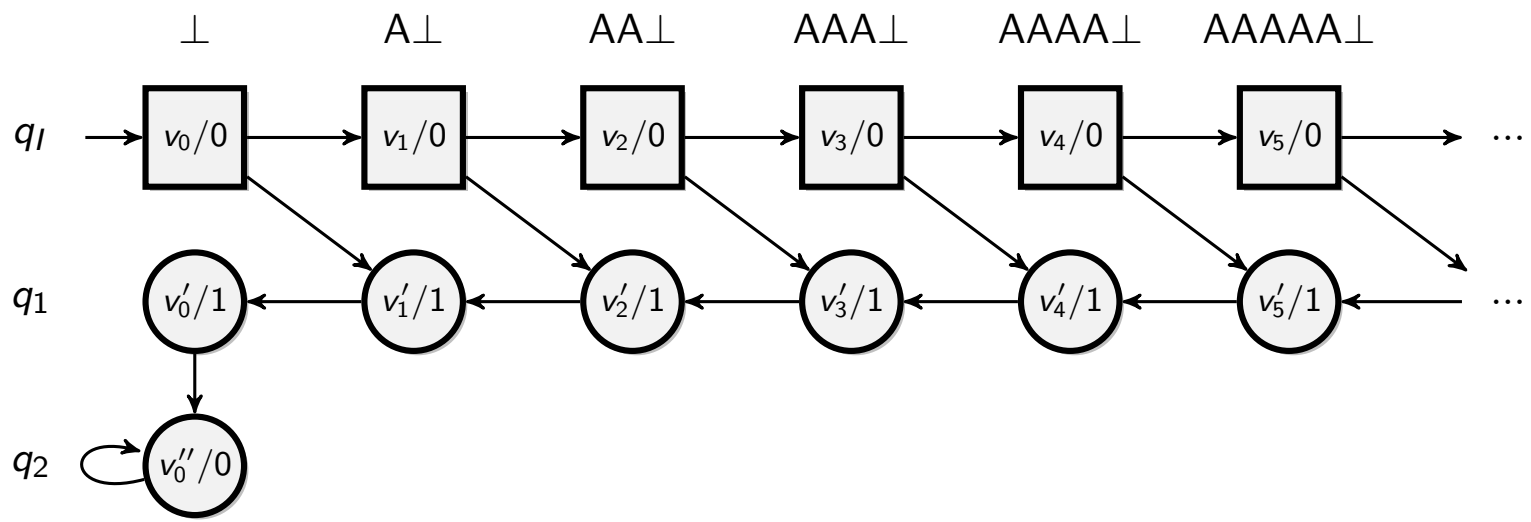
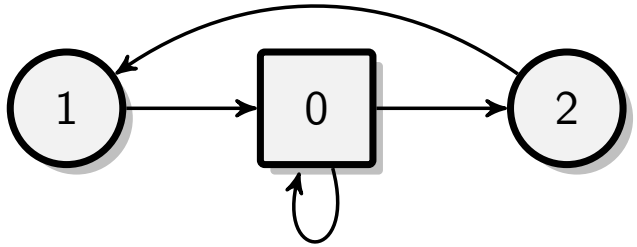


- Pushdown Parity Games can be reduced to parity games in exponentially sized arenas $\Rightarrow$ solvable in exponential time.
- Both players have positional winning strategies (but these are now infinite objects!).
- Finite representation of winning strategies: pushdown automata with output.

Quantitative Games

- Parity game: Player 0 wins from everywhere, but it takes arbitrarily long to "answer" 1 with 2.



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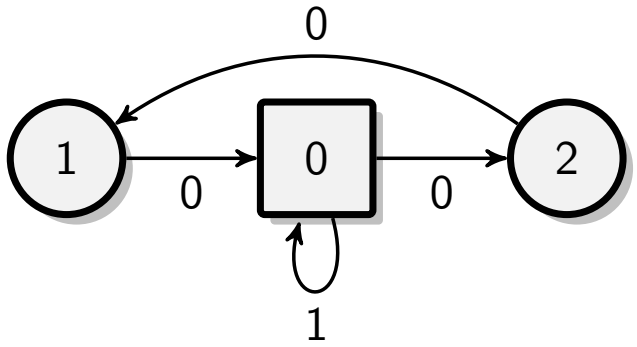

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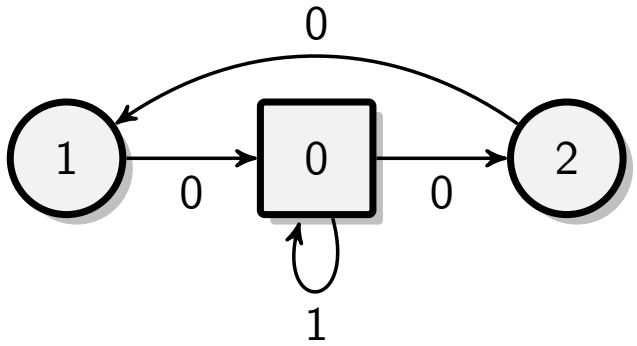

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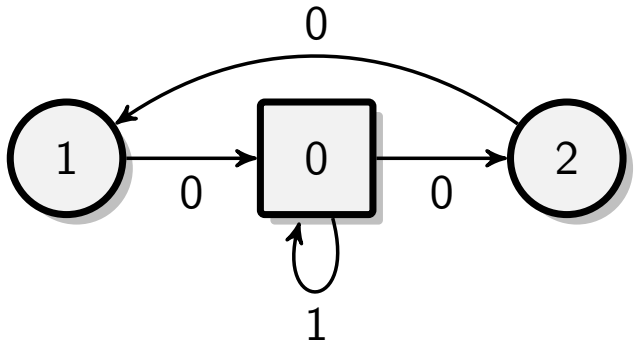

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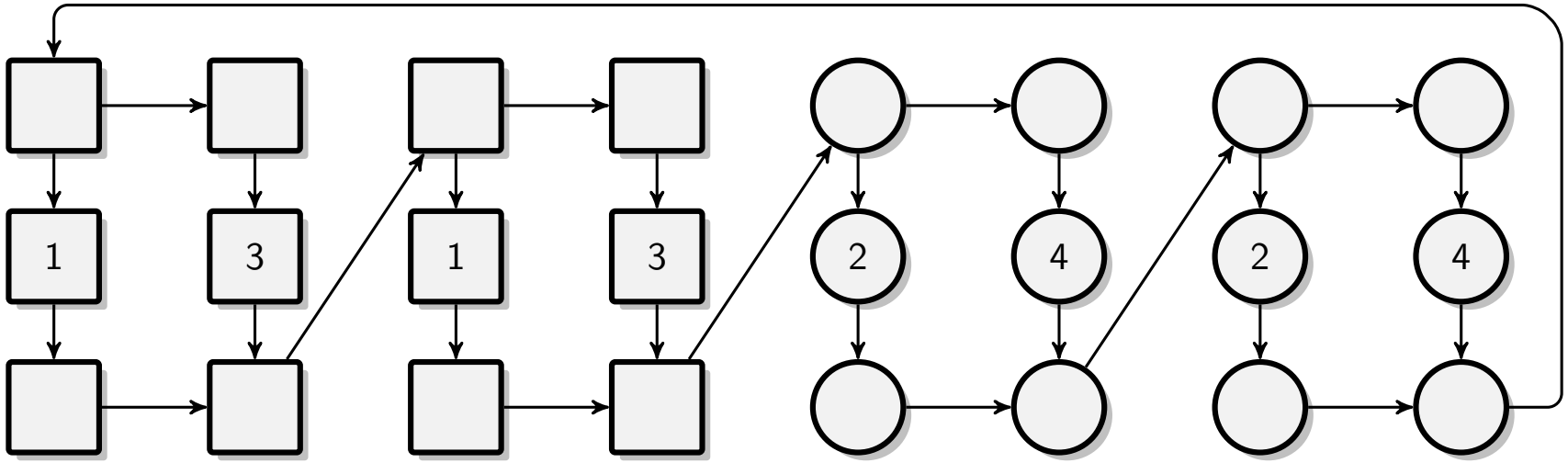
Theorem

Parity games with costs are determined, Player 0 *has positional winning strategies, and they can be solved in quasi-polynomial time.*

Tradeoffs

Every edge has cost 1



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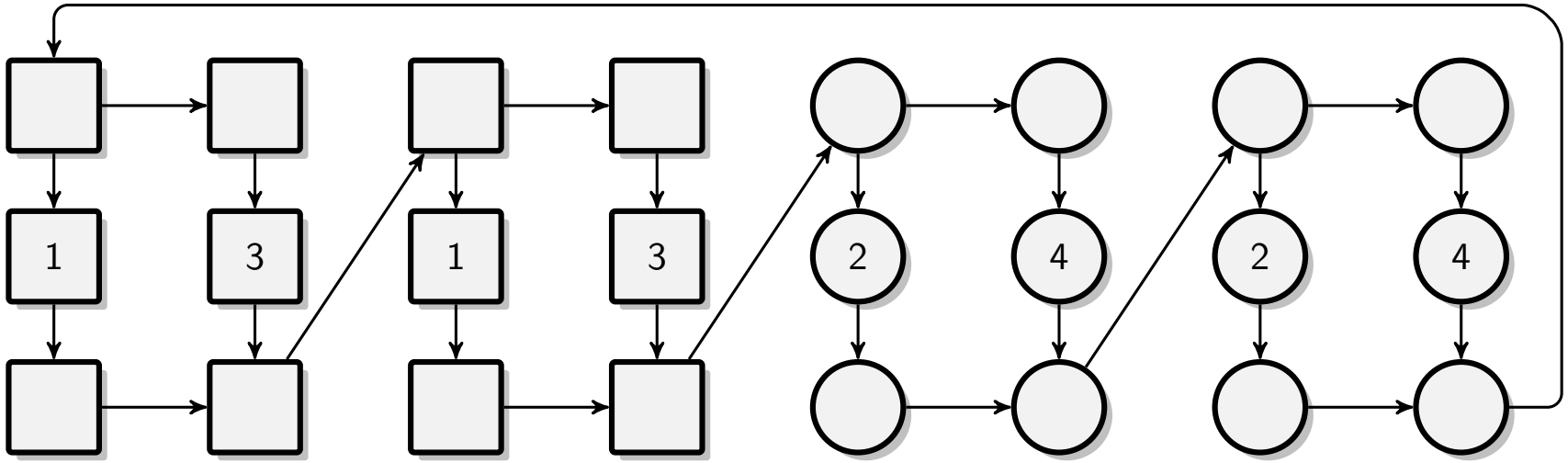

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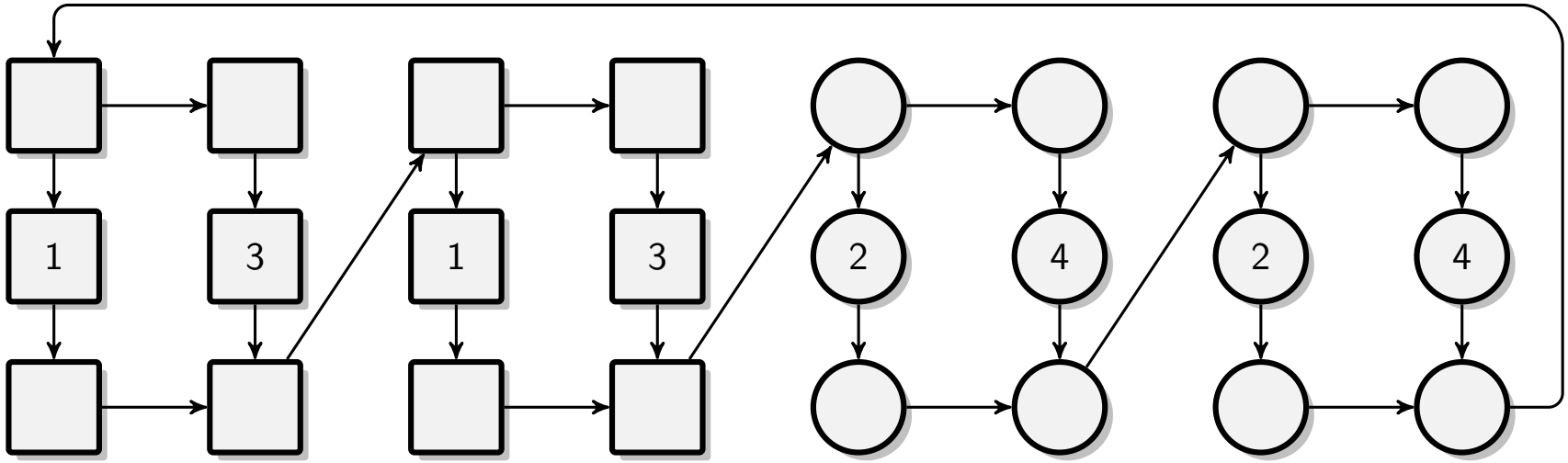

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With d odd colors and d gadgets for each player: Player 0 has:

- Positional winning strategy with bound $d^2 + 3d - 1$.
- Finite-state strategy with 2^{d-1} states and bound $d^2 + 2d$.

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And: any combination of extensions discussed above.

Distributed Synthesis

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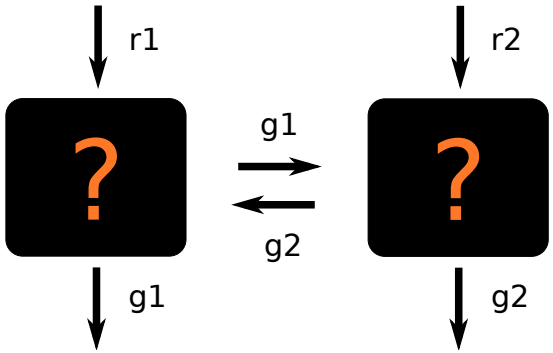

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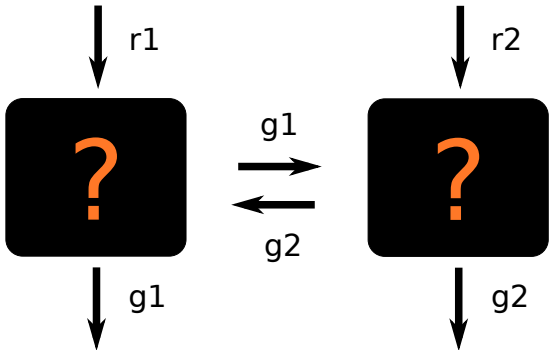

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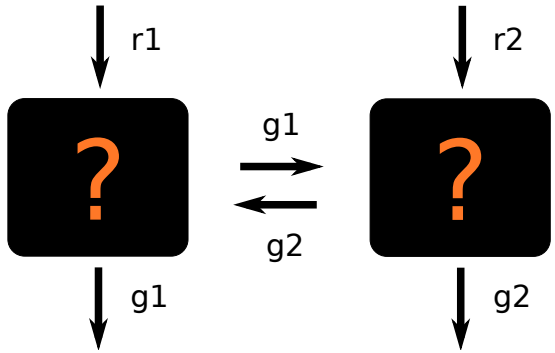

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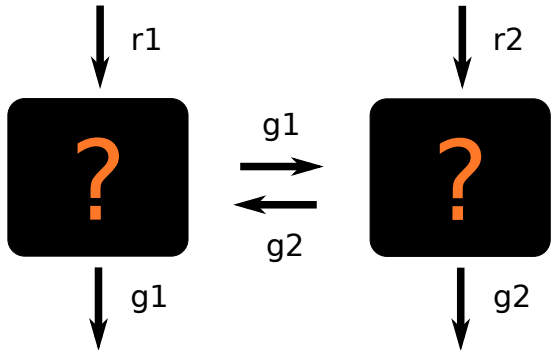

And: combinations with previously mentioned extensions, as well as special properties in distributed systems.

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- Byzantine faults: a subset of the components could be controlled by an adversary
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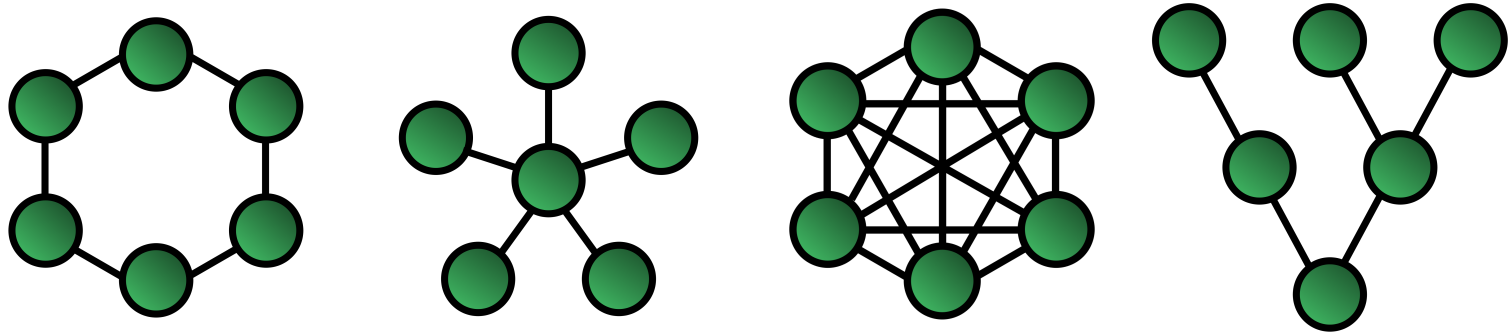
$\Rightarrow$ semi-decision procedures



Thank you
&
Good luck for the exam!