

Learning Assumptions for Compositional Verification

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Lecture 2

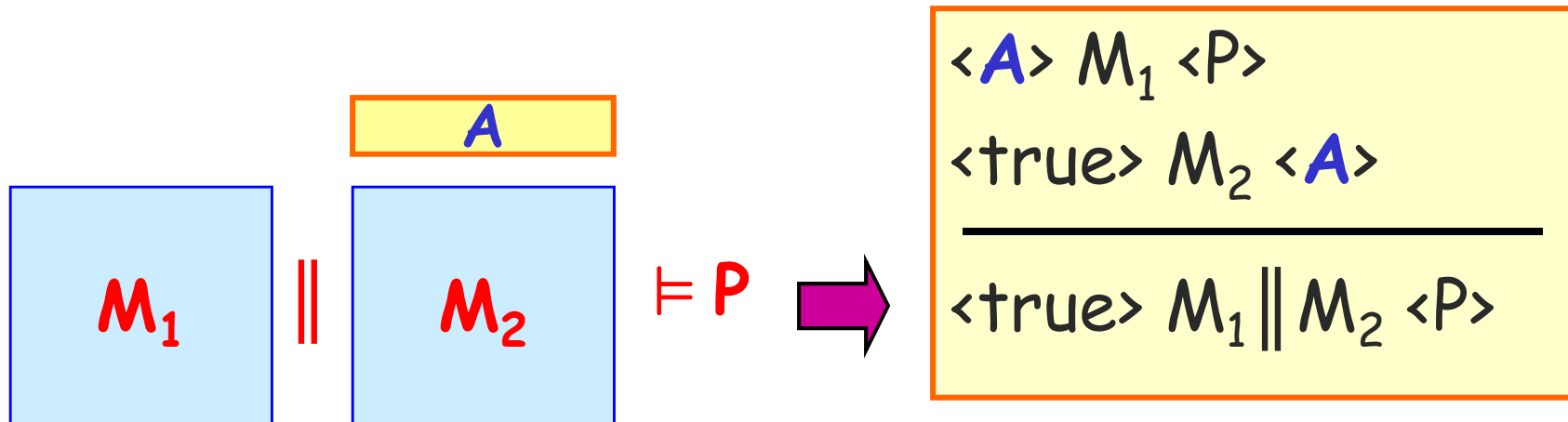
Compositional Verification

Basic building block:

$\langle A \rangle M \langle P \rangle$: whenever M is part of a system satisfying the **assumption** A , then the system must also **guarantee** P

Useful AG Rule for Safety Properties

1. check if a component M_1 guarantees P when it is a part of a system satisfying assumption A .
2. discharge assumption: show that the remaining component M_2 (the environment) satisfies A .



Assume-Guarantee

$$\frac{\langle A \rangle M_1 \langle P \rangle \quad \langle \text{true} \rangle M_2 \langle A \rangle}{\langle \text{true} \rangle M_1 \parallel M_2 \langle P \rangle}$$

- Crucial element: **assumption** A.
- Has to be **strong enough** to eliminate violations of P, but also **general enough** to reflect the environment M_2 appropriately.
- requires non-trivial human input in defining assumptions.

How to automatically construct assumptions ?

Labeled Transition Systems (LTS)

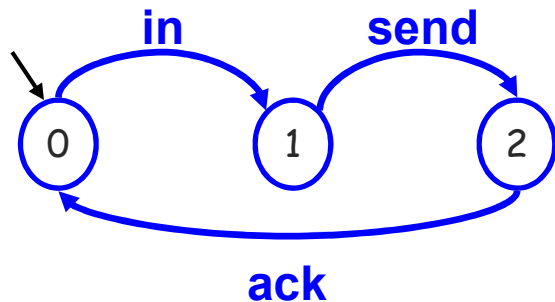
- Act - set of **observable actions**
 - LTSs communicate using observable actions
- τ - **local** \ internal action

LTS $M = (Q, q^0, \alpha M, \delta)$

- Q : finite non-empty set of **states**
- $q^0 \in Q$: **initial state**
- $\alpha M \subseteq \text{Act}$: observable actions
- $\delta \subseteq Q \times (\alpha M \cup \{\tau\}) \times Q$: **transition relation**

Alphabet

Labeled Transition Systems (LTS)



Traces:

$\langle \text{in} \rangle, \langle \text{in}, \text{send} \rangle, \dots$

- **Trace** of an LTS M : finite sequence of **observable** actions occurring on a computation, starting at the **initial state**.
- $L(M)$ = the **Language** of M : the set of all traces of M .

Parallel Composition $M_1 \parallel M_2$

- Components **synchronize** on **common observable** actions (communication).
- The remaining actions are **interleaved**.

$$M_1 = (Q_1, q^0_1, \alpha M_1, \delta_1), M_2 = (Q_2, q^0_2, \alpha M_2, \delta_2)$$

$$\rightarrow M_1 \parallel M_2 = (Q, q^0, \alpha M, \delta)$$

- $Q = Q_1 \times Q_2$
- $q^0 = (q^0_1, q^0_2)$
- $\alpha M = \alpha M_1 \cup \alpha M_2$

Transition Relation

Synchronization on $a \in \alpha M_1 \cap \alpha M_2$:

$(q_1, a, q_1') \in \delta_1$ and $(q_2, a, q_2') \in \delta_2$:

$\rightarrow ((q_1, q_2), a, (q_1', q_2')) \in \delta$

Interleaving on $a \in (\alpha M \setminus (\alpha M_1 \cap \alpha M_2)) \cup \{\tau\}$:

• $(q_1, a, q_1') \in \delta_1$ and $a \notin \alpha M_2$:

$\rightarrow ((q_1, q_2), a, (q_1', q_2)) \in \delta$ for any $q_2 \in Q_2$

• $(q_2, a, q_2') \in \delta_2$ and $a \notin \alpha M_1$:

$\rightarrow ((q_1, q_2), a, (q_1, q_2')) \in \delta$ for any $q_1 \in Q_1$

Safety Properties

Also expressed as LTSs, but of a special kind:

Safety LTS :

- **Deterministic:**

- Does **not** contain τ -transitions, and
- every state has **at most one** outgoing transition for each action:

$$(q, a, q'), (q, a, q'') \in \delta \rightarrow q' = q''$$

Safety Properties

For a **safety LTS**, P :

- $L(P)$ describes the set of **legal** (acceptable) behaviors over αP .

Language Containment

$$L(M) \uparrow_{\alpha P} \subseteq L(P)$$

$$M \models P \text{ iff } \forall \sigma \in L(M) : (\sigma \uparrow_{\alpha P}) \in L(P)$$

Note that, since we check $M \models P$, $\alpha P \subseteq \alpha M$

Model Checking $M \models P$

Safety LTS $P \rightarrow$ an Error LTS, P_{err} :

- “traps” violations with special **error state** π .

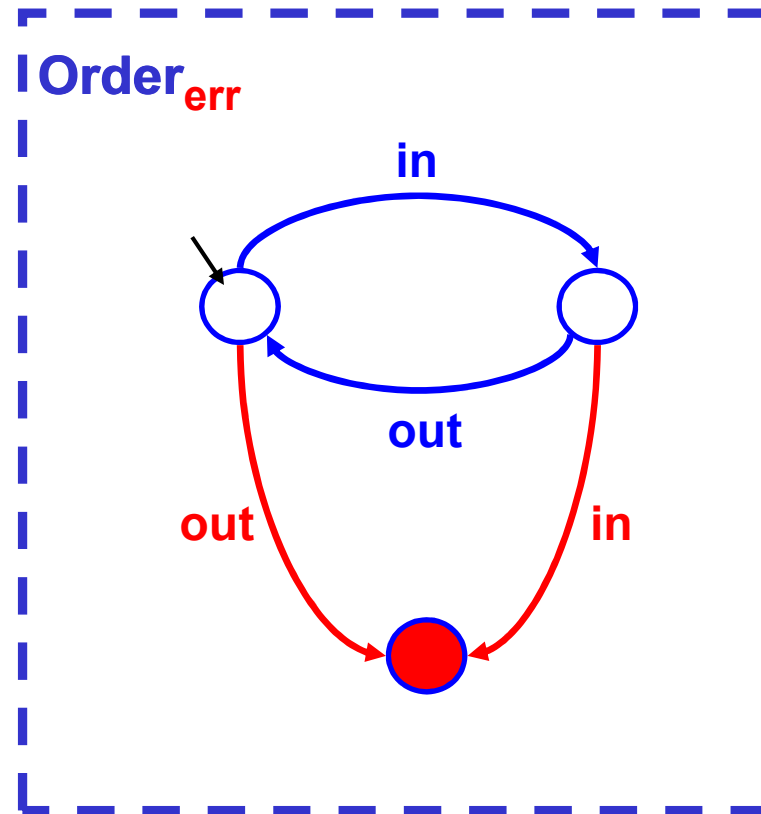
$$P = (Q, q^0, \alpha P, \delta)$$

$\rightarrow P_{err} = (Q \cup \{\pi\}, q^0, \alpha P, \delta')$, where:

$$\delta' = \delta \cup \{(q, a, \pi) \mid a \in \alpha P \text{ and } \nexists q' \in Q: (q, a, q') \in \delta\}$$

- Error LTS is **complete**.
- π is a deadend state: has no outgoing transitions.

Example



Model Checking $M \models P$

In automata:

$M \models \varphi$ iff

$$L(A_M \cap \bar{A}_\varphi) = \emptyset$$

Theorem:

- $M \models P$ iff π is unreachable in $M \parallel P_{err}$

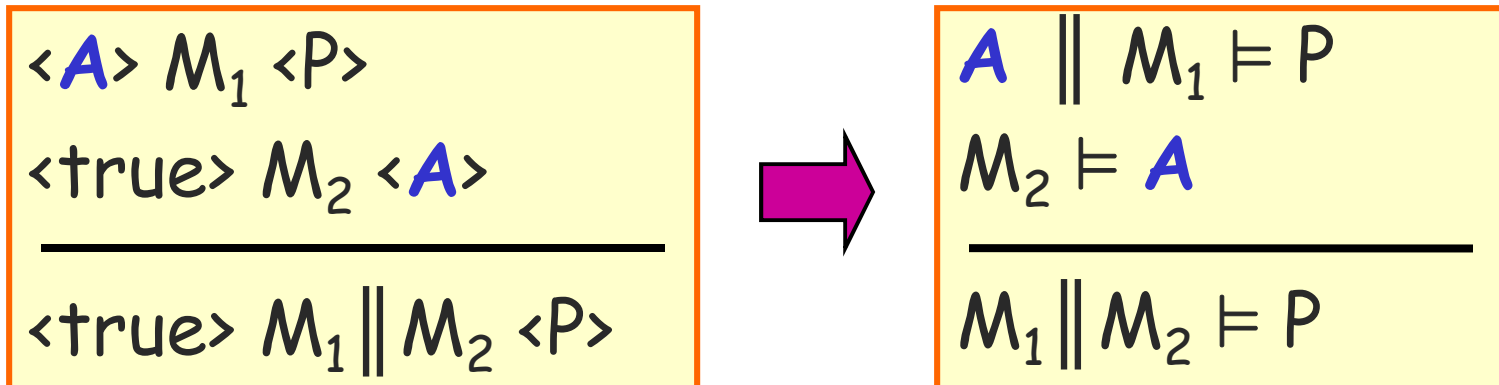
Recall that,

- $M \models P$ iff $\forall \sigma \in L(M) : (\sigma \uparrow \alpha P) \in L(P)$
- $M \parallel P_{err}$ synchronizes on αP

End summary of lecture 1

Assume Guarantee Reasoning

- **Assumptions**: also expressed as **safety LTSs**.
- $\langle A \rangle M \langle P \rangle$ is true iff $A \parallel M \models P$
i.e. π is unreachable in $A \parallel M \parallel P_{\text{err}}$



$M_1, M_2 : \text{LTSSs}$
 $P, A : \text{safety LTSSs}$

Outline

- ✓• Motivation
- ✓• Setting
 - Automatic Generation of Assumptions for the AG Rule
 - Learning algorithm
 - Assume-Guarantee with Learning
 - Example

Important concept: Weakest assumption for Σ

Definition: Let

M_1 - LTS component

P - safety property

Σ - interface alphabet of M_1 and environment

The **weakest assumption** A_w is a deterministic LTS such that:

- $\alpha A_w = \Sigma \cup (\alpha P - \alpha M_1)$
- For every M'_2 such that $\alpha A_w \subseteq \alpha M'_2$
 $\langle \text{true} \rangle M_1 \parallel M'_2 \langle P \rangle$ iff $\langle \text{true} \rangle M'_2 \langle A_w \rangle$

Note that, $\langle \text{true} \rangle M_1 \parallel A_w \langle P \rangle$ holds

Weakest assumption for M_1 , M_2 , P

- Interface alphabet for M_1 and M_2 and P :

$$\Sigma_I = (\alpha M_1 \cup \alpha P) \cap \alpha M_2$$

- The **weakest assumption** for M_1 , M_2 and P is defined as the weakest assumption for Σ_I

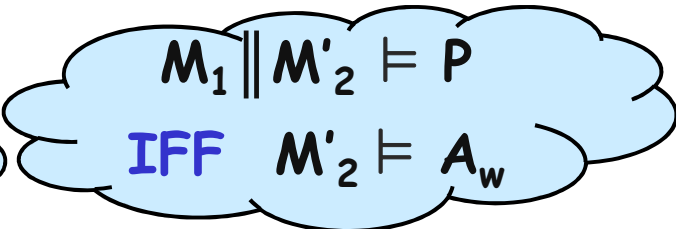
Observation 1

$$\frac{A \parallel M_1 \models P \quad M_2 \models A}{M_1 \parallel M_2 \models P}$$

- AG Rule will return **conclusive** results with the **Weakest assumption A_w** under which M_1 satisfies P :
 - Under assumption A_w on M_1 's environment, M_1 satisfies P , i.e. $A_w \parallel M_1 \models P$
 - **Weakest**: for all env. M'_2 : $M_1 \parallel M'_2 \models P$ **IFF** $M'_2 \models A_w$
- Given A_w , to check if $M_1 \parallel M_2 \models P$, check if M_2 meets the assumption A_w .

sufficient and **necessary** assumption on M_1 's env.

How to Obtain A_w ?


$$M_1 \parallel M'_2 \models P$$
$$\text{IFF } M'_2 \models A_w$$

- Given module M_1 , property P and M_1 's interface with its environment $\Sigma = (\alpha M_1 \cup \alpha P) \cap \alpha M_2$:

A_w describes exactly **all traces** σ over Σ such that in the context of σ , M_1 satisfies P

- Expressible as a safety LTS
- Can be computed algorithmically

Drawback of using A_w : if the computation runs out of memory, no assumption is obtained.

Observation 2

$$\begin{array}{l} A \parallel M_1 \models P \\ M_2 \models A \\ \hline M_1 \parallel M_2 \models P \end{array}$$

- No need to use the **weakest** env. assumption A_w .
 - AG rule might be applicable with **stronger** (less general) assumption.
- Instead of finding A_w :
- Use **learning** algorithm to **learn** A_w .
 - Use candidates A_i produced by learning algorithm as **candidate assumptions**: try to apply AG rule with A_i .

Given a Candidate
Assumption A_i

$$\begin{array}{l} A \parallel M_1 \models P \\ M_2 \models A \\ \hline M_1 \parallel M_2 \models P \end{array}$$

If $A_i \parallel M_1 \models P$ does **not** hold:

Assumption A_i is not tight enough

→ need to **strengthen** A_i

Given a Candidate Assumption A_i

$$\begin{array}{l} A \parallel M_1 \models P \\ M_2 \models A \\ \hline M_1 \parallel M_2 \models P \end{array}$$

Suppose $A_i \parallel M_1 \models P$ holds:

If $M_2 \models A_i$ also holds $\rightarrow M_1 \parallel M_2 \models P$ **verified!**

Otherwise: $\sigma \in L(M_2)$ but $(\sigma \uparrow \alpha \Sigma) \notin L(A_i)$ -- **cex**
 P violated by M_1 in the context of **cex** = $(\sigma \uparrow \alpha \Sigma)$?

Yes: real violation !

$\rightarrow M_1 \parallel M_2 \models P$ **falsified !**

No: spurious violation.

\rightarrow need to find better approximation of A_w .

"cex $\parallel M_1$ " $\models P$?

Given a Candidate Assumption A_i

$$\begin{array}{l} A \parallel M_1 \models P \\ M_2 \models A \\ \hline M_1 \parallel M_2 \models P \end{array}$$

Note:

If $A_i \parallel M_1 \models P$ does **not** hold:

→ traces are **removed** from A_i

If $M_2 \models A_i$ does **not** hold, and cex is spurious:

→ traces are **added** to A_i

Changes to A_i 's are non-monotonic but we get "closer" to A_w

" $cex \parallel M_1$ " $\models P$?

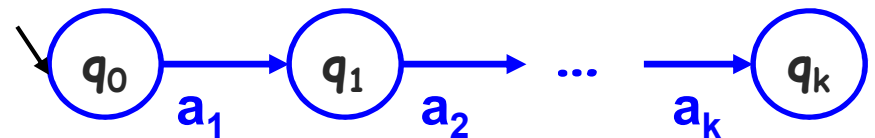
" $cex \in L(A_w)$ " ?

Compose M_1 with LTS A_{cex} over $\alpha\Sigma$

$cex = a_1, \dots, a_k \rightarrow A_{cex}$:

$Q = \{q_0, \dots, q_k\}$, $q^0 = q_0$

$= \{ (q_i, a_{i+1}, q_{i+1}) \mid 0 \leq i < k \}$



Model check $A_{cex} \parallel M_1 \models P$

Is π reachable in $A_{cex} \parallel M_1 \parallel P_{err}$?

yes

Real violation

no

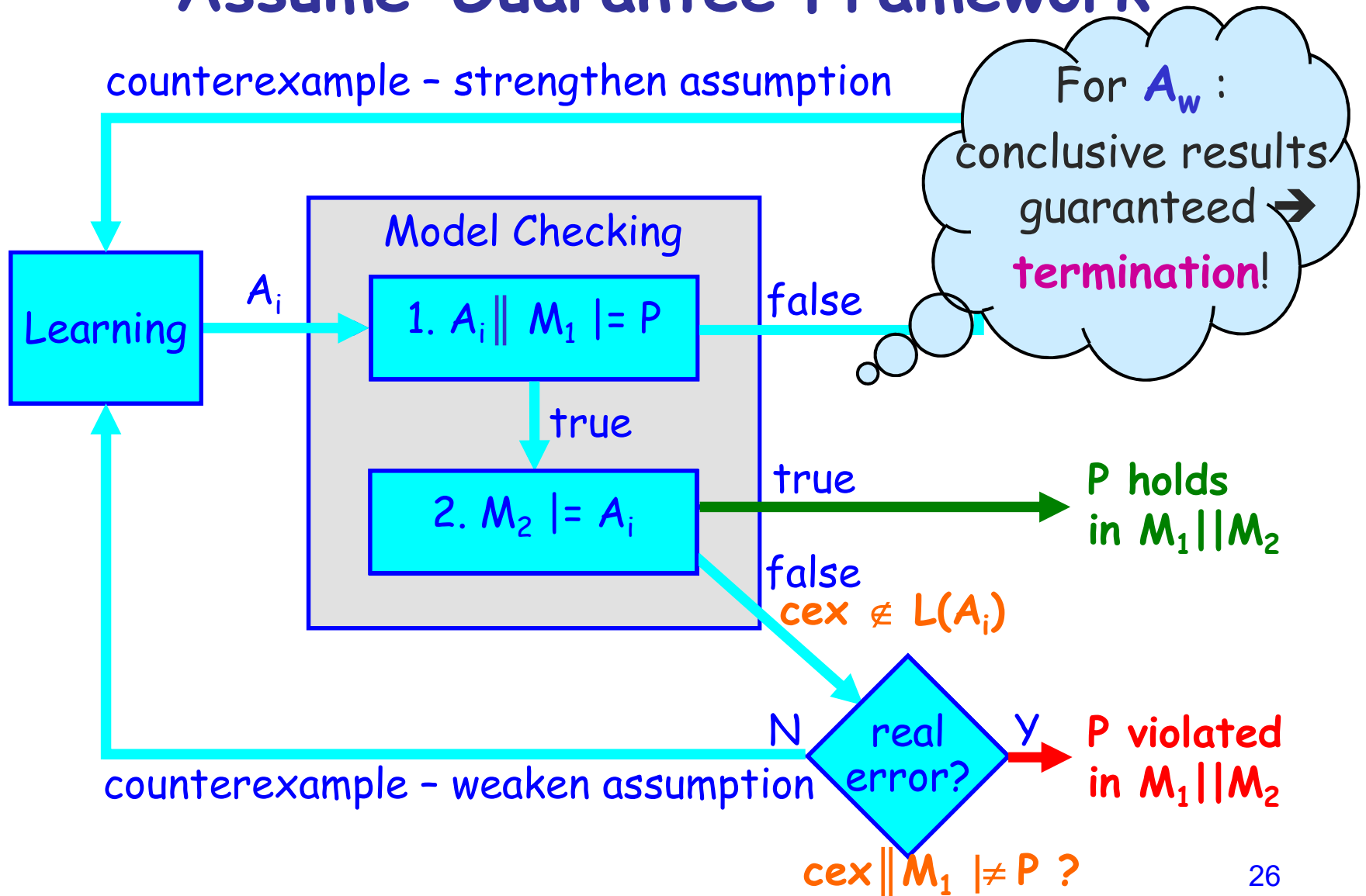
Spurious cex

Alternatively: simulate cex on $M_1 \parallel P_{err}$

$cex \parallel M_1 \models P$ iff when cex is simulated on $M_1 \parallel P_{err}$

it cannot lead to π (error) state.

Assume-Guarantee Framework



Outline

- ✓ Motivation
- ✓ Setting
- ✓ Automatic Generation of Assumptions for the AG Rule
 - Learning algorithm
 - Assume-Guarantee with Learning
 - Example

Learning Algorithm for DFA - L^*

- L^* : by Angluin, improved by Rivest & Schapire
- learns an unknown regular language U
- produces a Deterministic Finite-state Automaton (DFA) C such that $L(C) = U$

DFA $M = (Q, q^0, \alpha_M, \delta, F)$:

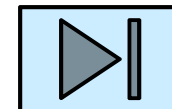
- Q, q^0, α_M, δ : as in deterministic LTS
- $F \subseteq Q$: accepting states
- $L(M) = \{\sigma \mid \delta(q^0, \sigma) \in F\}$

Learning Algorithm for DFA - L^*

- L^* interacts with a **Teacher** to answer two types of questions:
 - **Membership queries**: is string σ in U ?
 - **Conjectures**: for a candidate DFA C_i , is $L(C_i) = U$?
- answers are **(true)** or **(false + counterexample)**

Conjectures C_1, C_2, \dots converge to C

Equivalence
Queries



Outline

- ✓ Motivation
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Assume-Guarantee with Learning

Reminder:

- Use **learning** algorithm to **learn** A_w .
- Use candidates produced by learning as **candidate assumptions** A_i for AG rule.

In order to **use** L^* to produce assumptions A_i :

- Show that $L(A_w)$ is **regular**
- Translate **DFA** to **safety LTS** (assumption)
- Implement the **teacher**

$L(A_w)$ is Regular

- A_w is expressible as a **safety LTS**
- Translation of **safety LTS** A into **DFA** C :

$$A = (Q, q^0, \alpha M, \delta) \rightarrow$$

$$C = (Q, q^0, \alpha M, \delta, Q)$$

- There **exists** a **DFA** C s.t. $L(C) = L(A_w)$
- L^* can be used to learn a DFA that accepts $L(A_w)$

DFA to Safety LTS

Weakest assumption is prefix-closed

- DFAs C_i returned by L^* are **complete**, **minimal** and in this setting also **prefix-closed**:
 - if $\sigma \in L(C_i)$, then every **prefix** of σ is **also** in $L(C_i)$.
- C_i contains a single **non-accepting** state q_{nf} and **no accepting** state is reachable from q_{nf} .
- To get a **safety LTS** simply remove non-accepting state and all its ingoing transitions :
$$C_i = (Q \cup \{q_{nf}\}, q^0, \alpha M, \delta, Q) \rightarrow$$
$$A_i = (Q, q^0, \alpha M, \delta \cap (Q \times \alpha M \times Q))$$

Implementing the Teacher

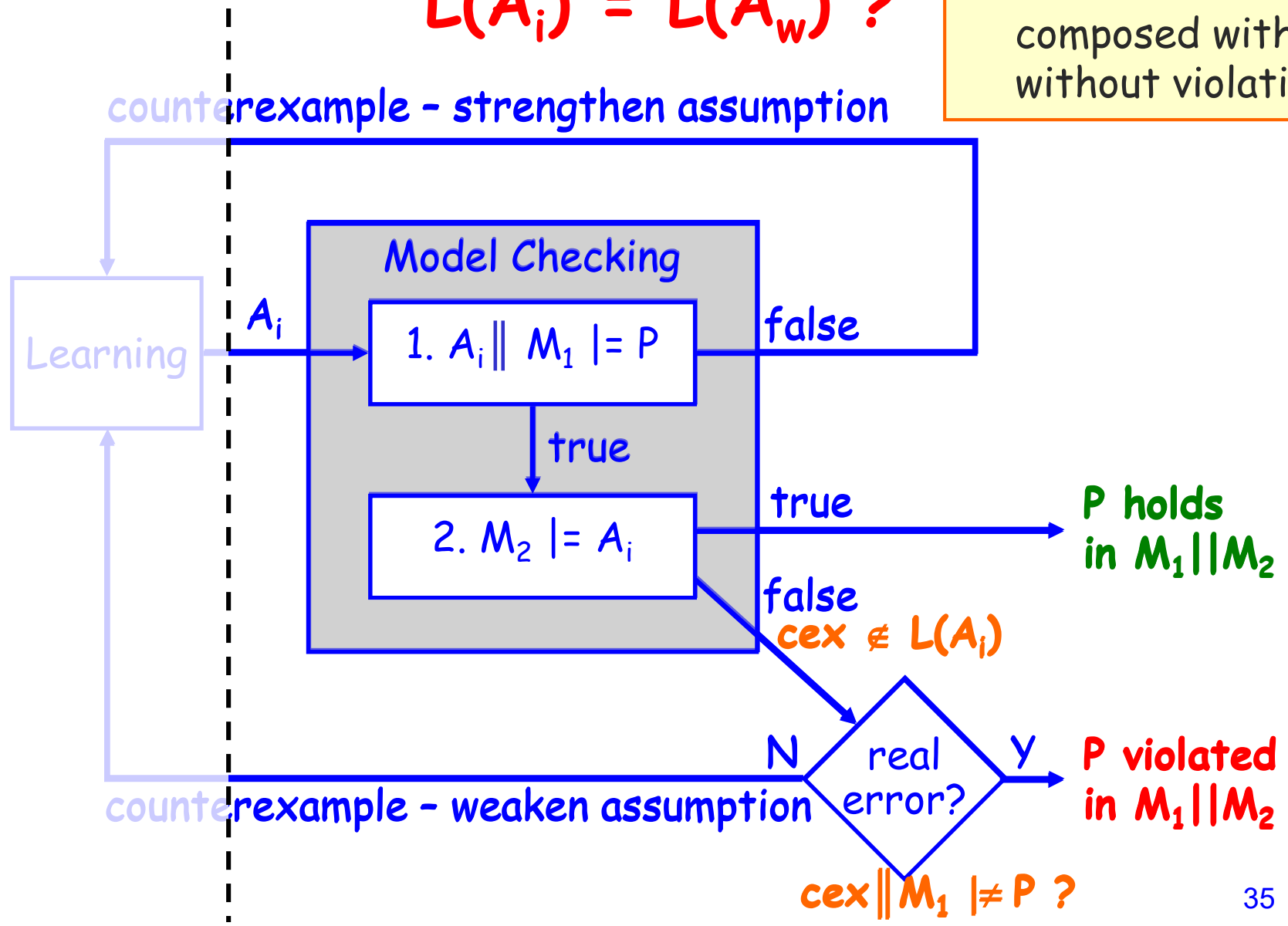
A_w is unknown...

$\sigma \in L(A_w)$ iff
in context of σ ,
 M_1 satisfies P

- **Membership** query: $\sigma \in L(A_w)$?
 - Check if " $\sigma \parallel M_1$ " $\models P$:
 - **Model checking**: is π reachable in $A_\sigma \parallel M_1 \parallel P_{err}$?
 - or - **Simulation**: is π (**error**) state reachable when simulating σ on $M_1 \parallel P_{err}$?
- **Equivalence** query: $L(C_i) = L(A_w)$?
 - Translate C_i into a safety LTS A_i
 - Use it as candidate assumption for **AG rule**

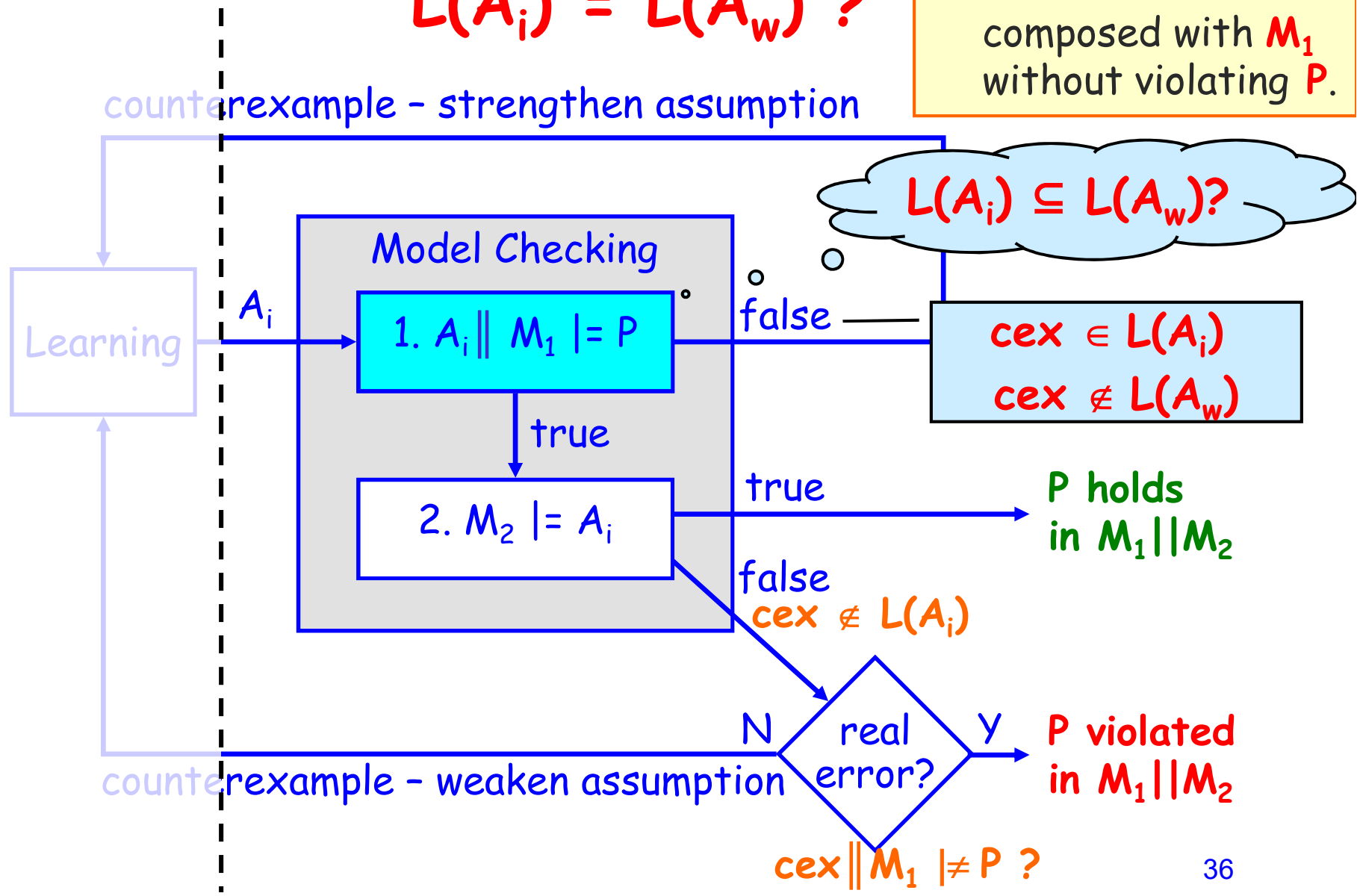
$$L(A_i) = L(A_w) ?$$

A_w contains all traces that can be composed with M_1 without violating P .



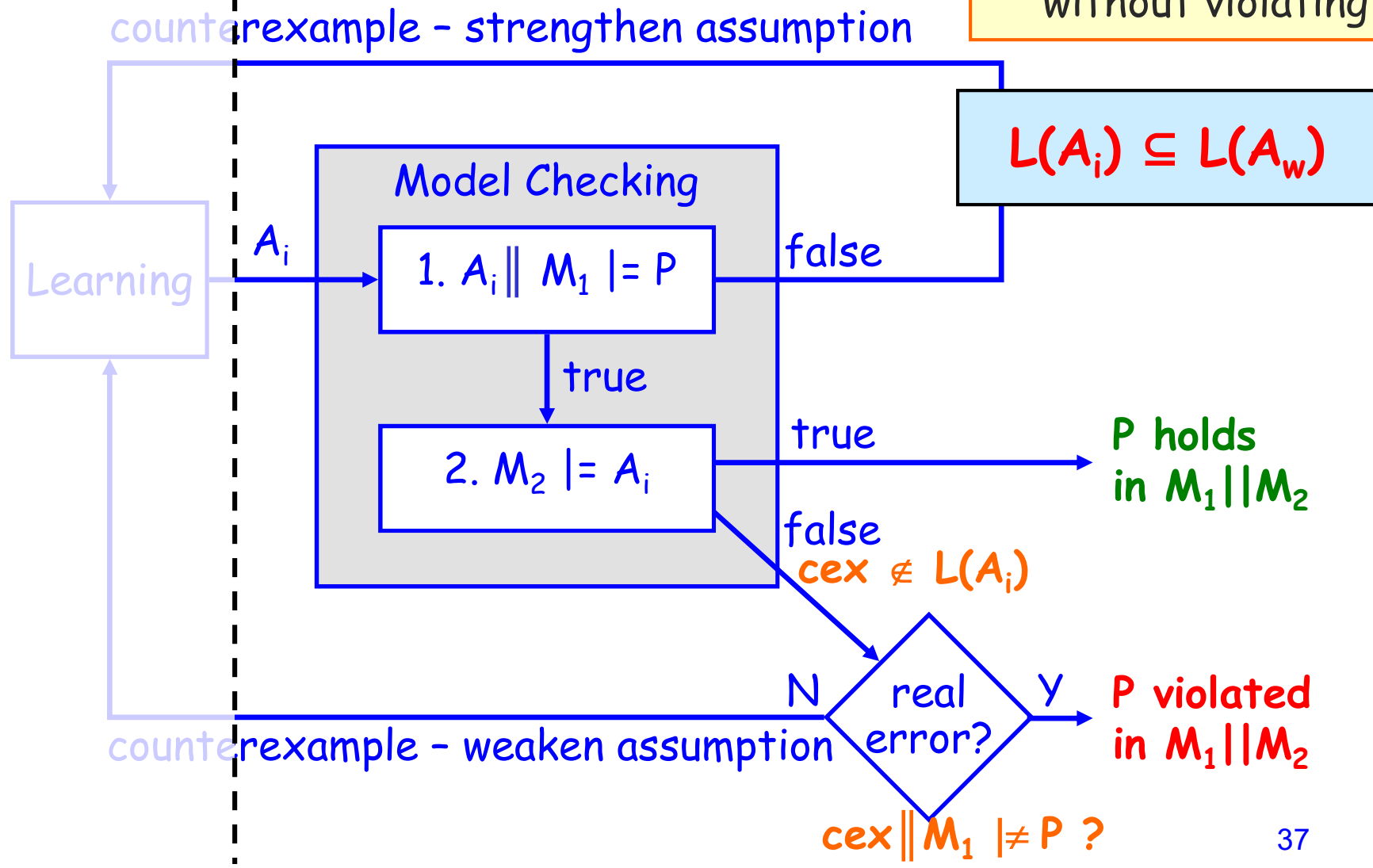
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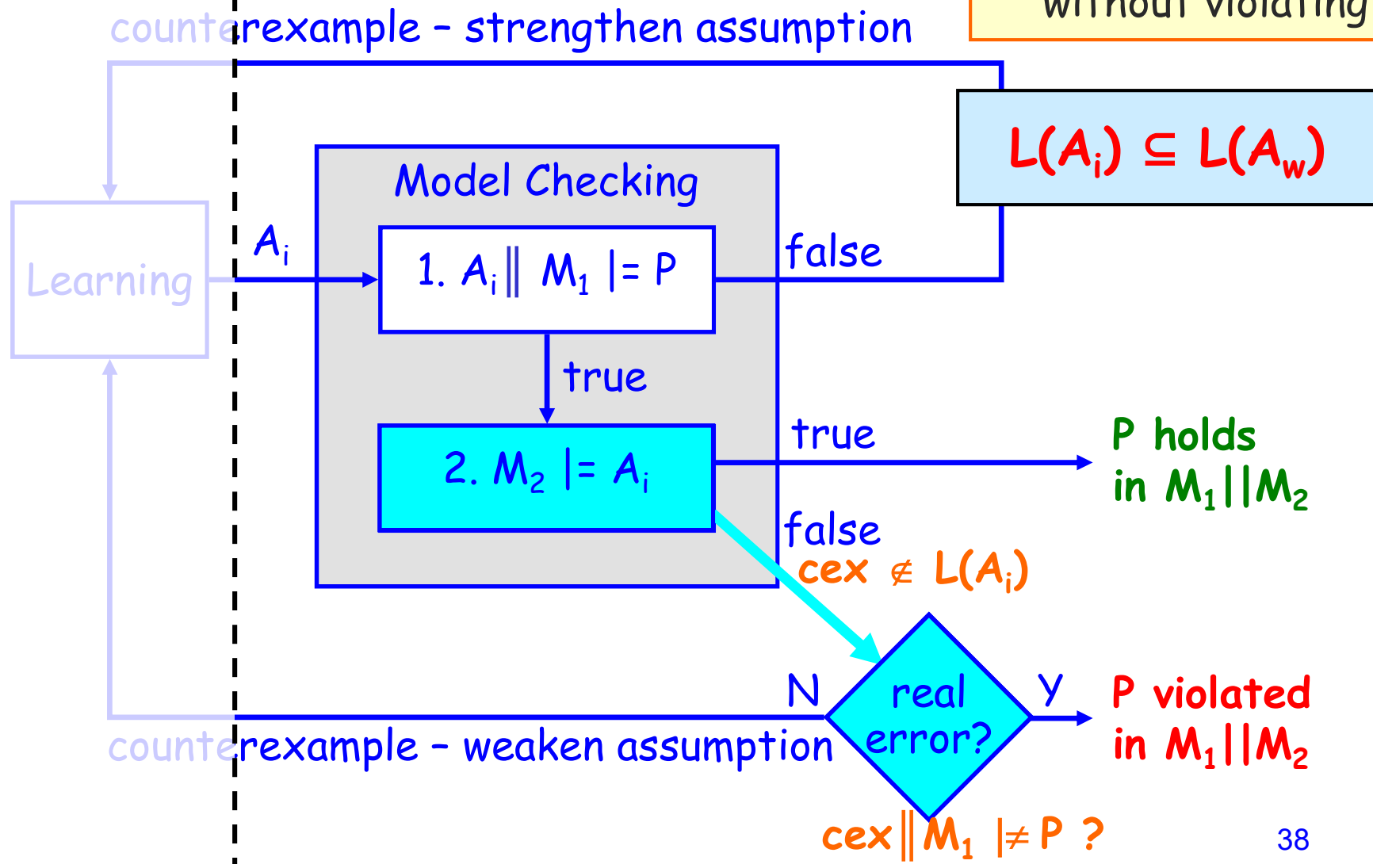
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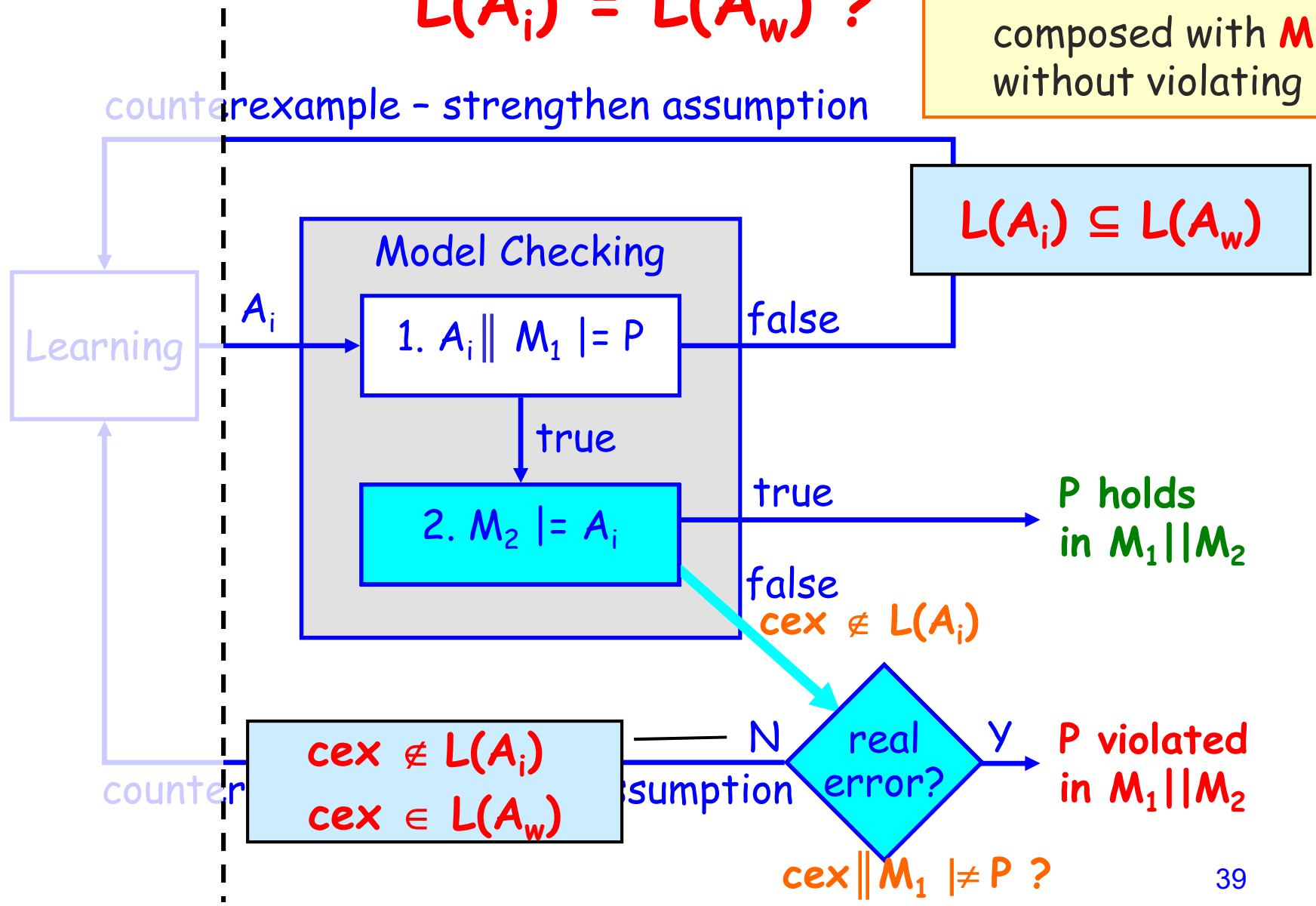
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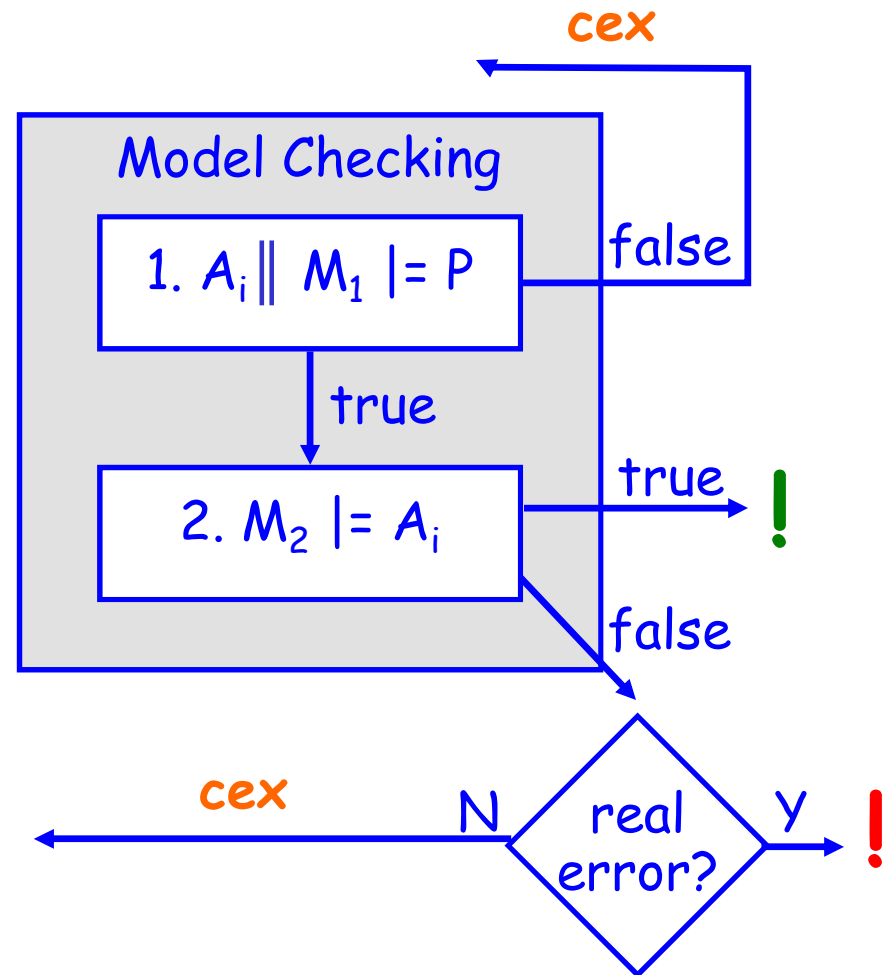
$L(A_i) = L(A_w) ?$

A_w contains all traces that can be composed with M_1 without violating P .



Equivalence Query - Summary

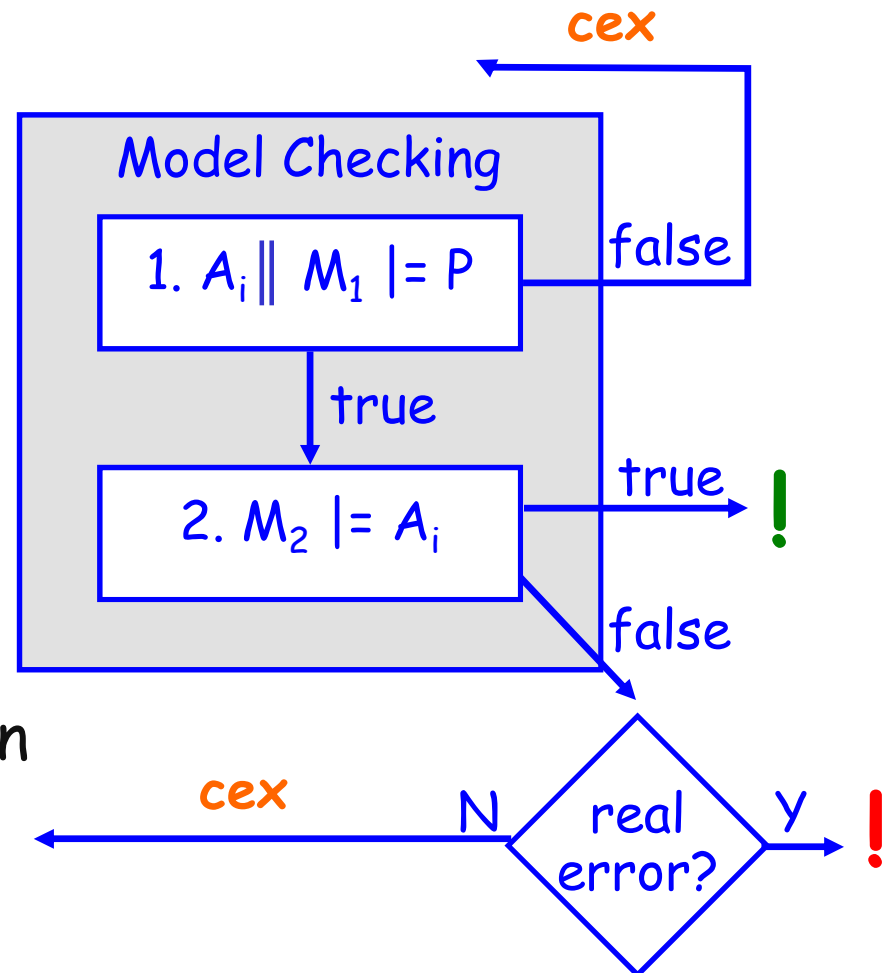
2 applications of
model checking
+ 1 **model checking**
or **simulation**



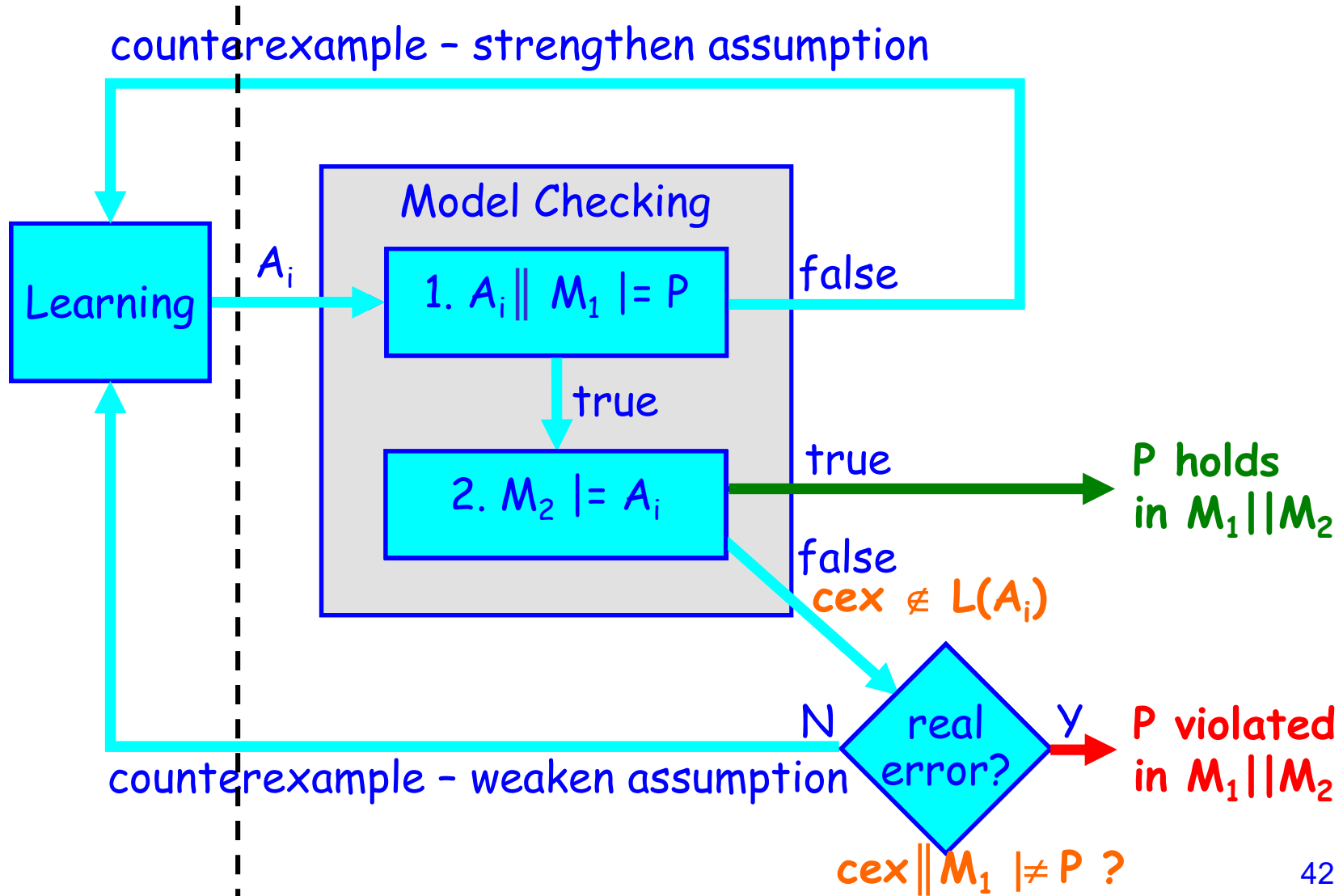
Equivalence Query - Summary

If $L(A_i) = L(A_w)$:
AG rule returns
conclusive results \rightarrow
No need to continue with L^*

If $L(A_i) \neq L(A_w)$:
Can terminate and abort L^* .
Otherwise, **cex** is returned in
1 if $L(A_i) \not\subseteq L(A_w)$, or
2 if $L(A_i) \not\supseteq L(A_w)$



Assume-Guarantee Framework



Characteristics of Framework

- **AG** uses conjectures produced by **L*** as candidate assumptions A_i
- **L*** uses **AG** as teacher
- **L*** terminates
 - Framework guaranteed to **terminate**:
 - **At latest** terminates when A_w is produced.
 - **Possibly** terminates **before** A_w is produced!

Characteristics of Framework

- L^* makes at most $n = |A_w|$ equivalence queries
- # membership queries: $O(kn^2 + n \log m)$
 - m : size of largest counterexample
 - k : size of Σ

or simulation

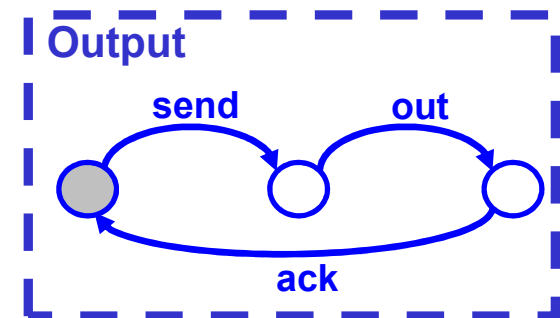
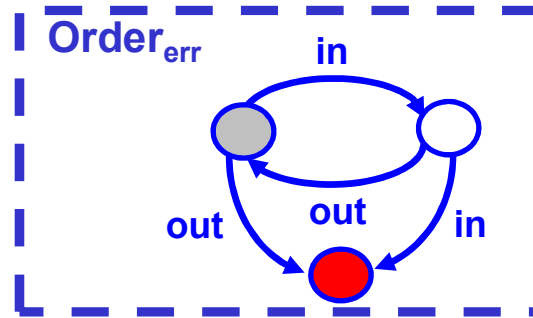
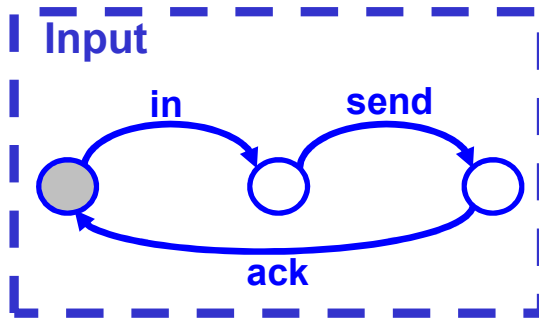
- membership query \rightarrow Model checking X 1
 - equivalence query \rightarrow Model checking X 3
- $\rightarrow O(kn^2 + n \log m)$ simulations
+ $O(n)$ model checking applications.

Each MC involves either M_1 or M_2 - not both!

Outline

- ✓ Motivation
- ✓ Setting
- ✓ Automatic Generation of Assumptions for the AG Rule
- ✓ Learning algorithm
- ✓ Assume-Guarantee with Learning
 - Example

Example



Check: $\text{Input} \parallel \text{Output} \models \text{Order} ?$

M_1

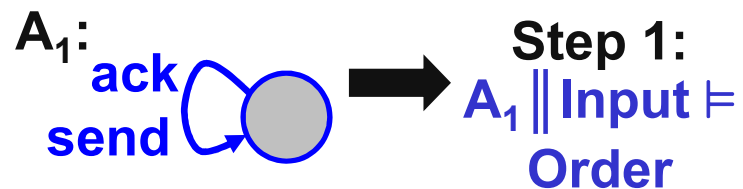
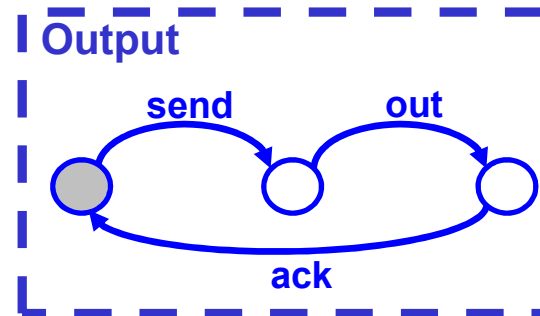
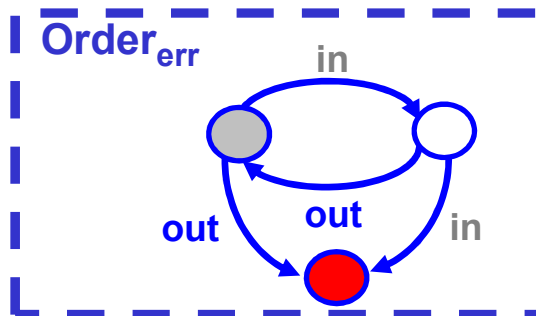
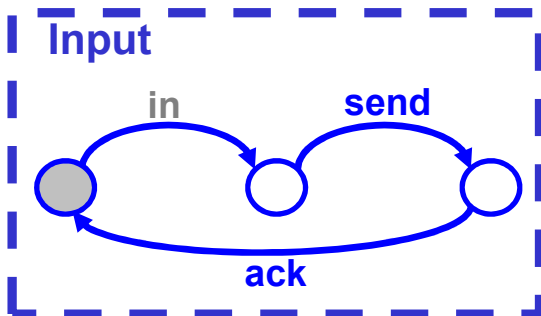
M_2

P

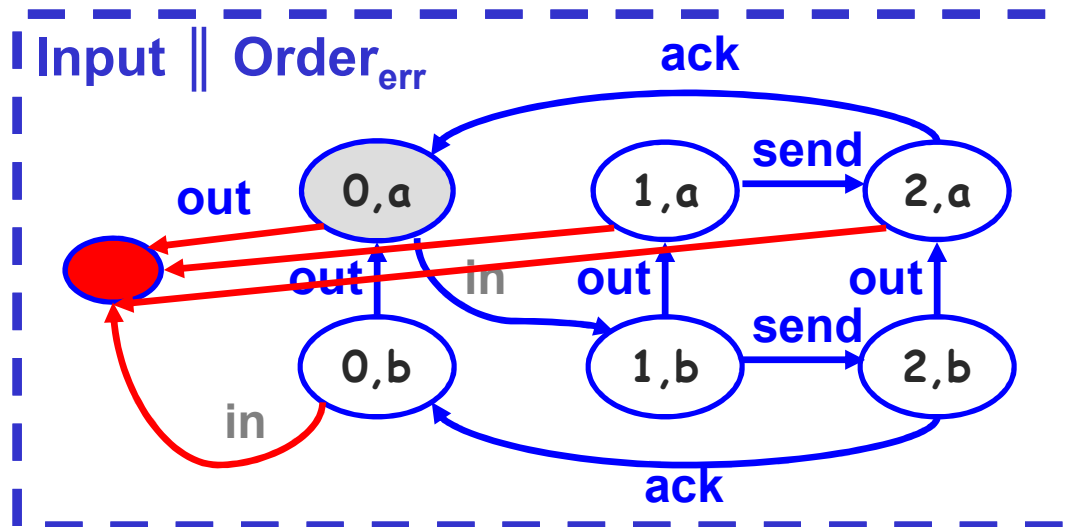
Σ (assumption's alphabet) : $\{\text{send, out, ack}\}$

$$\Sigma = (\alpha M_1 \cup \alpha P) \cap \alpha M_2$$

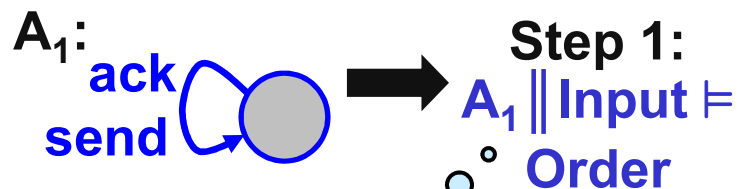
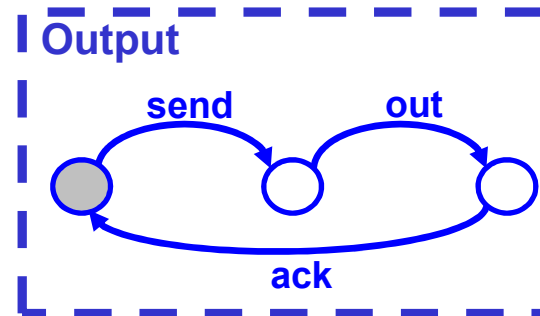
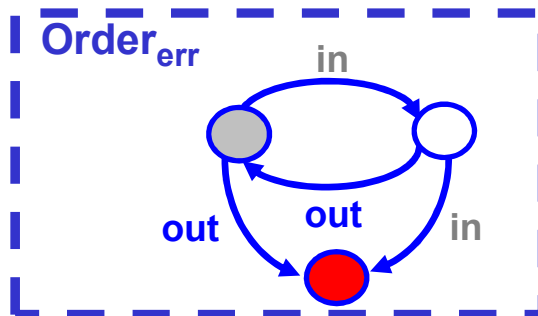
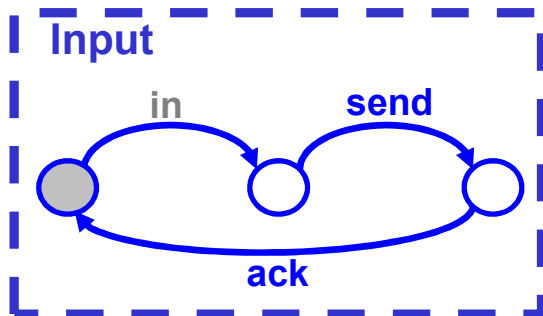
Conjectures



$\Sigma = \{\text{send, out, ack}\}$

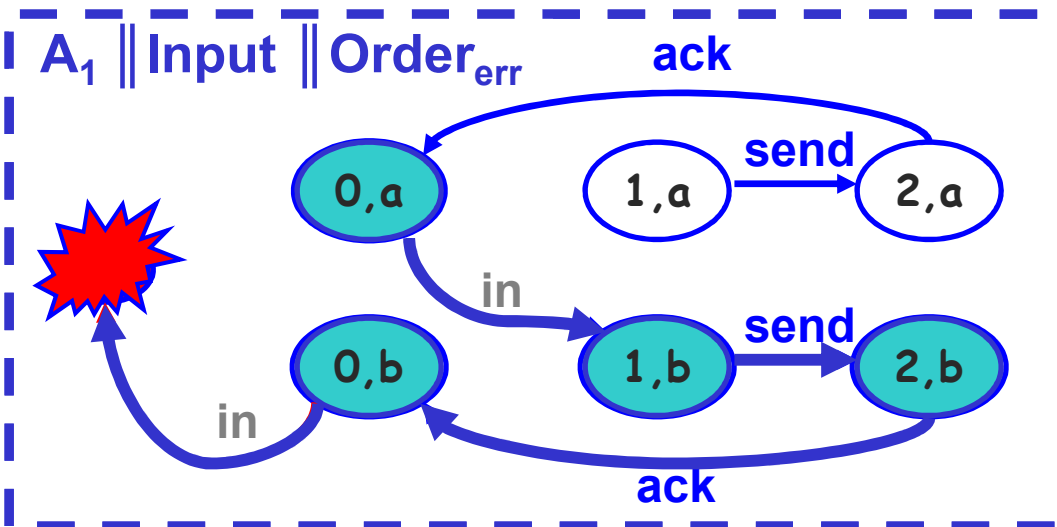


Conjectures

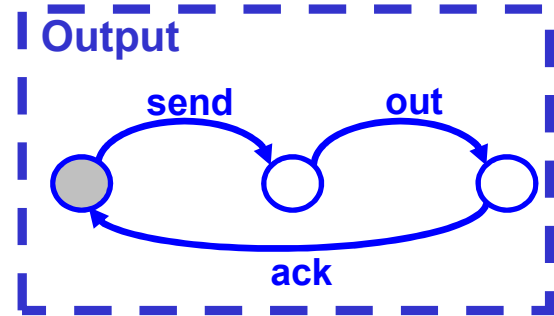
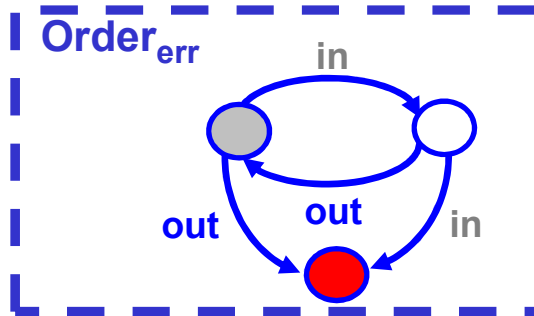
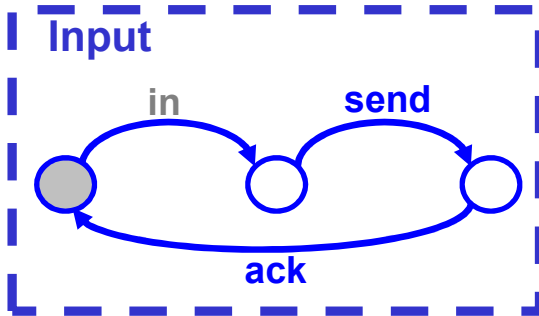


A₁ || Input ≡ Input
with $\Sigma = \{\text{in}, \text{send}, \text{ack}, \text{out}\}$

$\Sigma = \{\text{send}, \text{out}, \text{ack}\}$



Conjectures

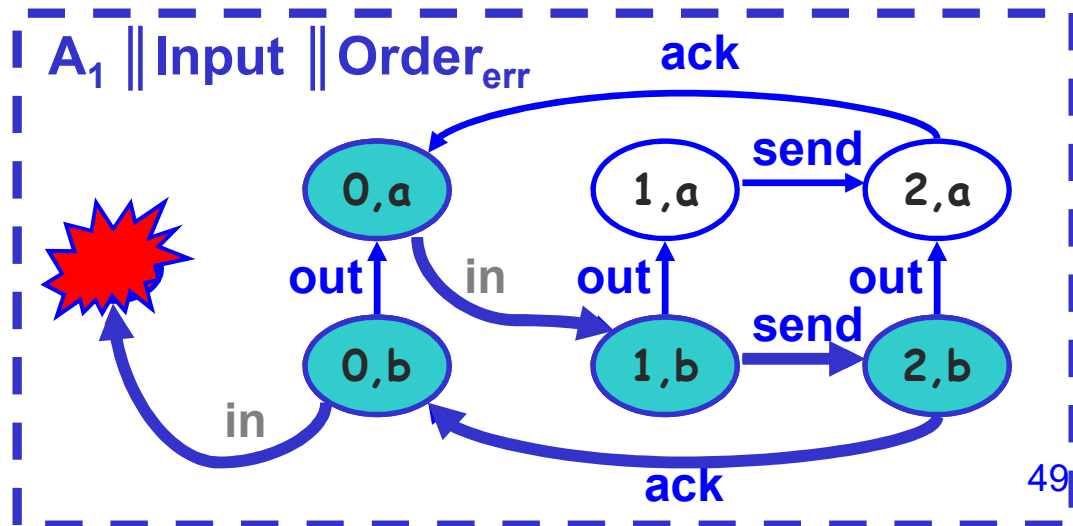


Step 1:
A₁ || Input |=
Order

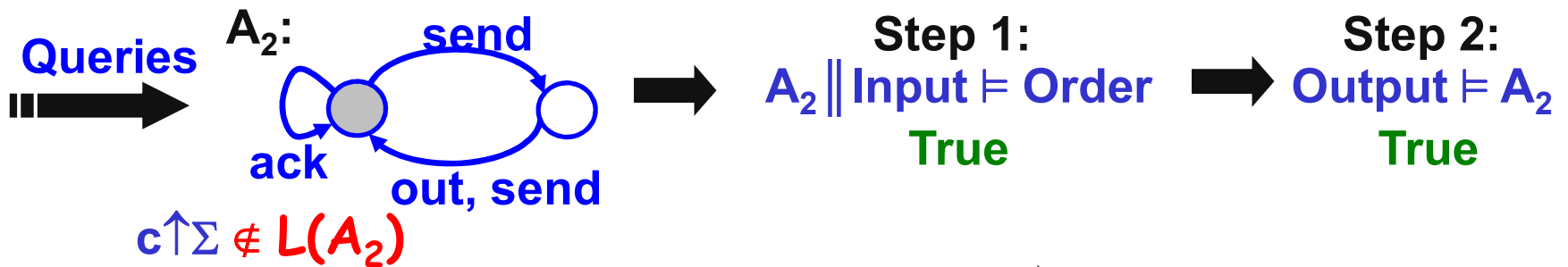
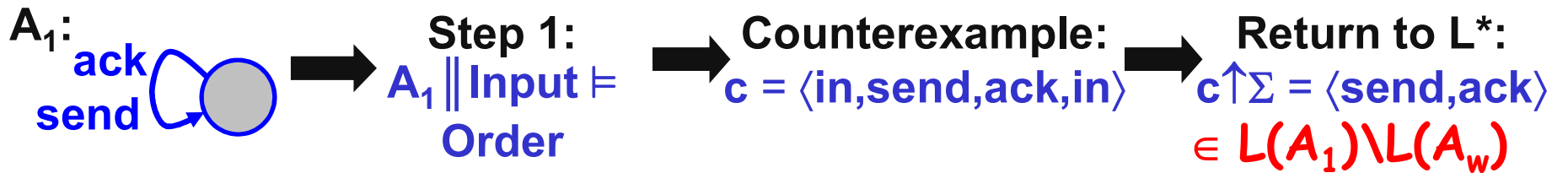
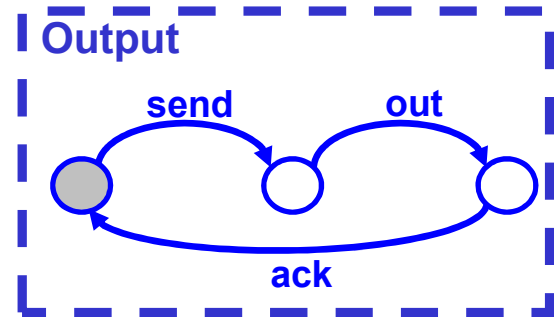
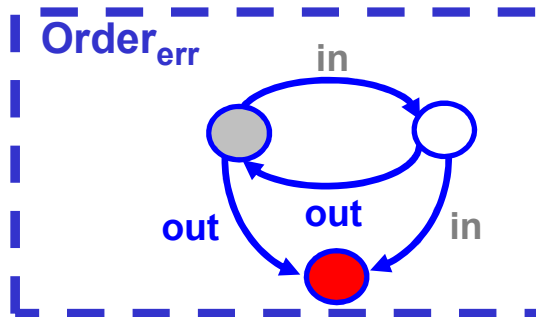
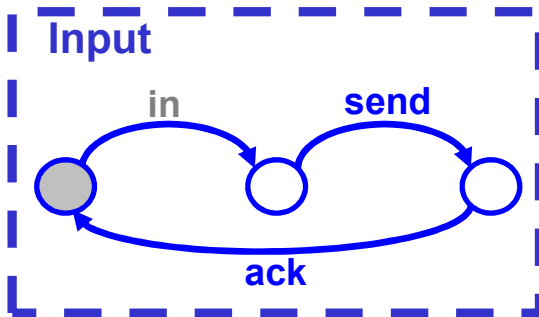
Counterexample:
c = ⟨in, send, ack, in⟩

Return to L*:
c↑Σ = ⟨send, ack⟩
∈ L(A₁) \ L(A_w)

Σ = {send, out, ack}



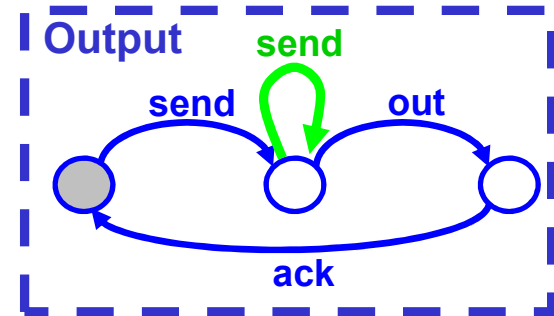
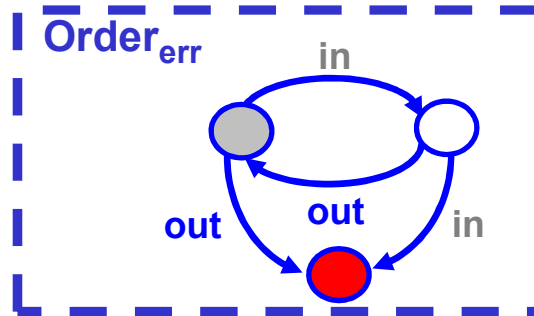
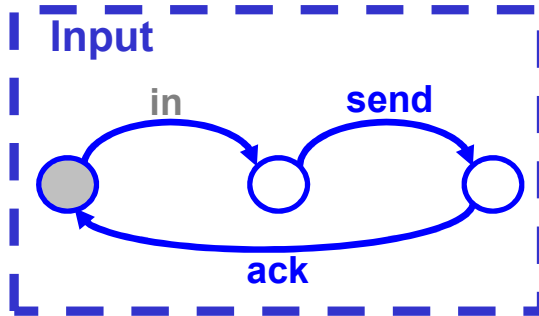
Conjectures



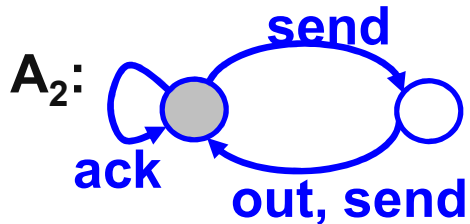
$\Sigma = \{\text{send}, \text{out}, \text{ack}\}$

\rightarrow property **Order** holds on $\text{Input} \parallel \text{Output}$

Another Example



Step 2: $\text{Output}' \models A_2$



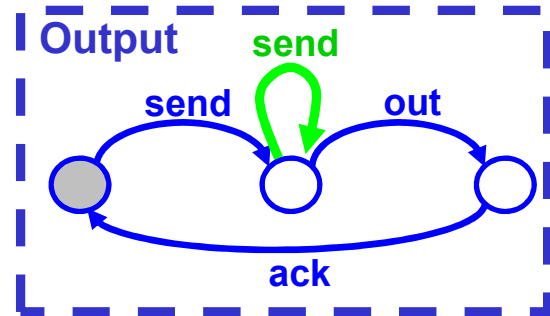
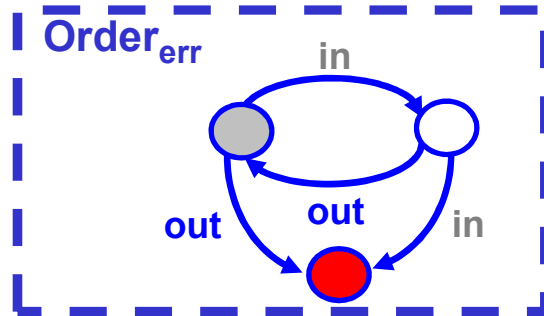
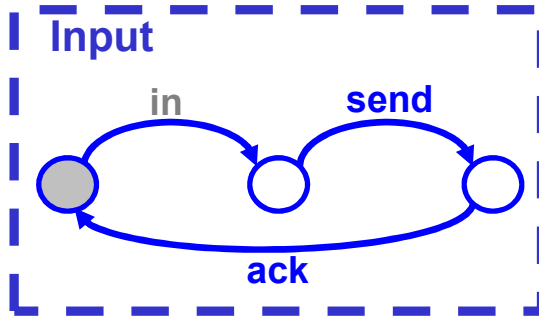
Counterexample: $c = \langle \text{send}, \text{send}, \text{out} \rangle$

real error?

Simulate $c \uparrow \Sigma = \langle \text{send}, \text{send}, \text{out} \rangle$ on $\text{Input} \parallel \text{Order}_{\text{err}}$

$\Sigma = \{\text{send}, \text{out}, \text{ack}\}$

Another Example



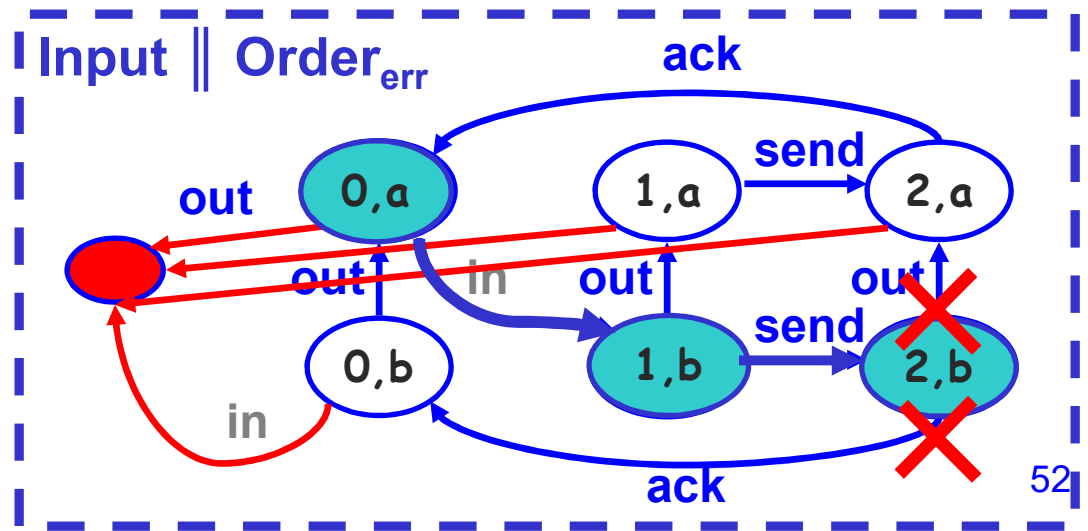
Step 2: $\text{Output}' \models A_2$

Counterexample: $c = \langle \text{send}, \text{send}, \text{out} \rangle$ **real error?**

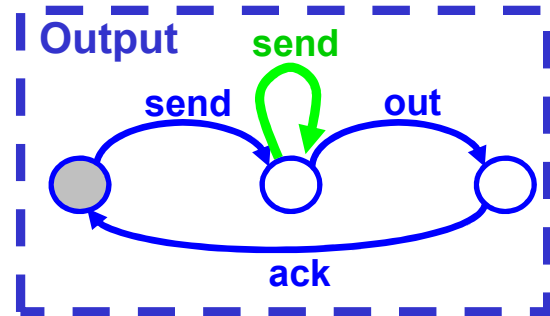
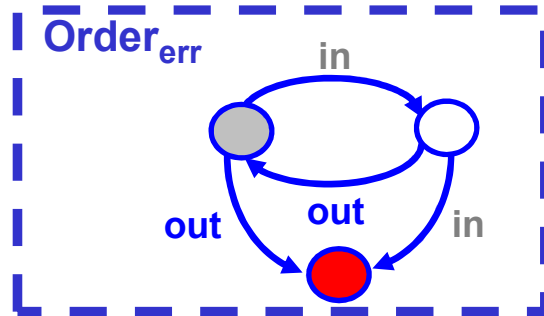
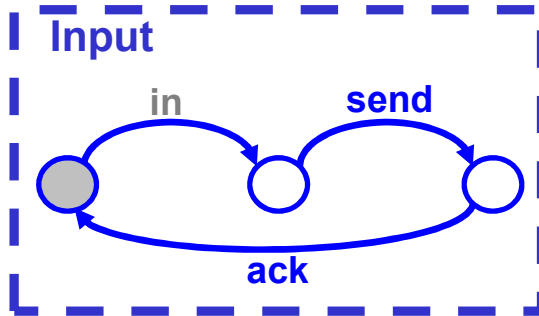
Simulate $c \uparrow \Sigma = \langle \text{send}, \text{send}, \text{out} \rangle$ on $\text{Input} \parallel \text{Order}_{\text{err}}$

π unreachable
not a real error!

$\Sigma = \{\text{send}, \text{out}, \text{ack}\}$



Another Example



Step 2: $\text{Output}' \models A_2$ \Rightarrow **Counterexample:** $c = \langle \text{send}, \text{send}, \text{out} \rangle$ $\xrightarrow{\text{real error?}}$ **Simulate** $c \uparrow \Sigma = \langle \text{send}, \text{send}, \text{out} \rangle$ **on** $\text{Input} \parallel \text{Order}_{\text{err}}$

π **unreachable**
not a real error!
 \Rightarrow **Return** $c \uparrow \Sigma$ **to** L^* \Rightarrow A_4 :

A_4 : Automaton with four states. Transitions: ack (top-left to top-right), send (top-right to middle), out (middle to top-right), ack (middle to bottom), send (bottom to middle), out (bottom to bottom), send (bottom to top-left).

\Rightarrow **property**
Order holds
on $\text{Input} \parallel \text{Output}'$

$\Sigma = \{\text{send}, \text{out}, \text{ack}\}$

A_w (in a thought bubble)

Conclusion

- Generate assumptions for assume-guarantee reasoning in an **incremental** and fully **automatic** fashion, using **learning**.
- Each iteration of **assume-guarantee** may conclude that the required property is **satisfied** or **violated** in the system.
- Assumption generation converges to an assumption that is **necessary** and **sufficient** for the property to hold in the specific system.

The End