Introduction to Software Verification

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Lectures Material
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Lecture 12

9.1.18

Summary

- Explicit model checking

State explosion problem

- BDD-based symbolic model checking
- SAT-based Bounded Model Checking (BMC)

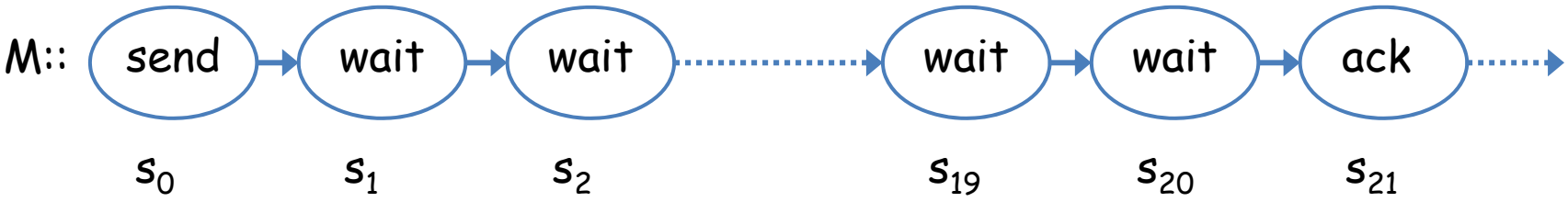
Other solutions to the state-explosion problem

Small models replace the full, concrete model:

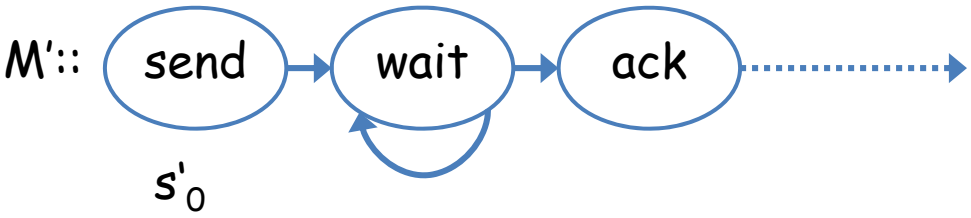
- Abstraction
- Compositional verification
- Partial order reduction
- Symmetry

example

Let M be a communication system in which there are exactly 20 wait steps between a send and an ack



M' includes all behaviors of M and more:



example

Every path in M has a "representative path" in M'. Therefore, if we prove:

$M', s_0' \models A(\neg ack \; W \; send)$

We can conclude that also:

$M, s_0 \models A(\neg ack \; W \; send)$

example

Since M' has more paths, if

$M', s'_0 \not\models AG(send \rightarrow F\ ack)$

then we cannot conclude that

$M, s_0 \not\models AG(send \rightarrow F\ ack)$

- A counterexample might be spurious
- Refinement might be needed

Equivalences and preorders

Goal: to define

- Preorder between models: $M_2 \geq M_1$ s.t.
  $M_2 \models \varphi \Rightarrow M_1 \models \varphi$

- Equivalence between models: $M_1 \equiv M_2$ s.t.
  $M_1 \models \varphi \Leftrightarrow M_2 \models \varphi$

Which properties are preserved?

We define:

equivalence between models that strongly preserves CTL*:

- If $M_1 \equiv M_2$ then for every CTL* formula φ,
  $M_1 \models \varphi \Leftrightarrow M_2 \models \varphi$

preorder between models that weakly preserves ACTL*

- If $M_2 \geq M_1$ then for every ACTL* formula φ,
  $M_2 \models \varphi \Rightarrow M_1 \models \varphi$

ACTL /ACTL*

- No existential path quantifier (no E)
  - Only A
- Negation is applied to atomic propositions only
- Need $\vee$ and $\wedge$
- U and the dual of U, V (release)

$M,\pi \models (f_1 \, V \, f_2) \Leftrightarrow \forall j \geq 0 \, [\, [\forall i < j. \, \pi^i \not\models f_1 \,] \Rightarrow \pi^j \models f_2 \,]$

- $f_1 V f_2 \equiv \neg (\neg f_1 \, U \, \neg f_2)$

ACTL

Universal CTL

- $p, \neg p,$ for $p \in AP$
- $g_1 \vee g_2,\ g_1 \wedge g_2$
- **AX** g_1, **A** (g_1 **U** g_2), **A**(g_1 **V** g_2)
  - **AG** g_1, **AF** g_1 (can be expressed by AU, AV)

where g_1, g_2 are ACTL (state formulas

Example : AG AF restart is an ACTL formula

The simulation preorder [Milner]

Given two models over AP:

$M_1 = (S_1, I_1, R_1, L_1)$, $M_2 = (S_2, I_2, R_2, L_2)$

$H \subseteq S_1 \times S_2$ is a **simulation** iff

for every $(s_1, s_2) \in H$:

- s_1 and s_2 satisfy the same propositions
- For every successor t_1 of s_1 there is a successor t_2 of s_2 such that $(t_1, t_2) \in H$

Notation:

$s_1 \leq s_2$ if there is simulation H, s.t. $(s_1, s_2) \in H$

The simulation preorder [Milner]

Given two models over AP:

$M_1 = (S_1, I_1, R_1, L_1)$, $M_2 = (S_2, I_2, R_2, L_2)$

$H \subseteq S_1 \times S_2$ is a **simulation** iff

for every $(s_1, s_2) \in H$:

- $L_1(s_1) = L_2(s_2)$

- $\forall t_1 [(s_1, t_1) \in R_1 \Rightarrow \exists t_2 [(s_2, t_2) \in R_2 \wedge (t_1, t_2) \in H]]$

Notation: $s_1 \leq s_2$

Simulation preorder (cont.)

$H \subseteq S_1 \times S_2$ is a **simulation** from M_1 to M_2 iff

H is a simulation and

for every $s_1 \in I_1$ there is $s_2 \in I_2$
s.t. $(s_1, s_2) \in H$

Notation: $M_1 \leq M_2$

Bisimulation relation [Park]

For models M_1 and M_2 over AP,

$B \subseteq S_1 \times S_2$ is a **bisimulation**

iff for every $(s_1, s_2) \in B$:

- $L_1(s_1) = L_2(s_2)$
- $\forall t_1 [(s_1, t_1) \in R_1 \Rightarrow \exists t_2 [(s_2, t_2) \in R_2 \wedge (t_1, t_2) \in B]]$
- $\forall t_2 [(s_2, t_2) \in R_2 \Rightarrow \exists t_1 [(s_1, t_1) \in R_1 \wedge (t_1, t_2) \in B]]$

Notation: $s_1 \equiv s_2$

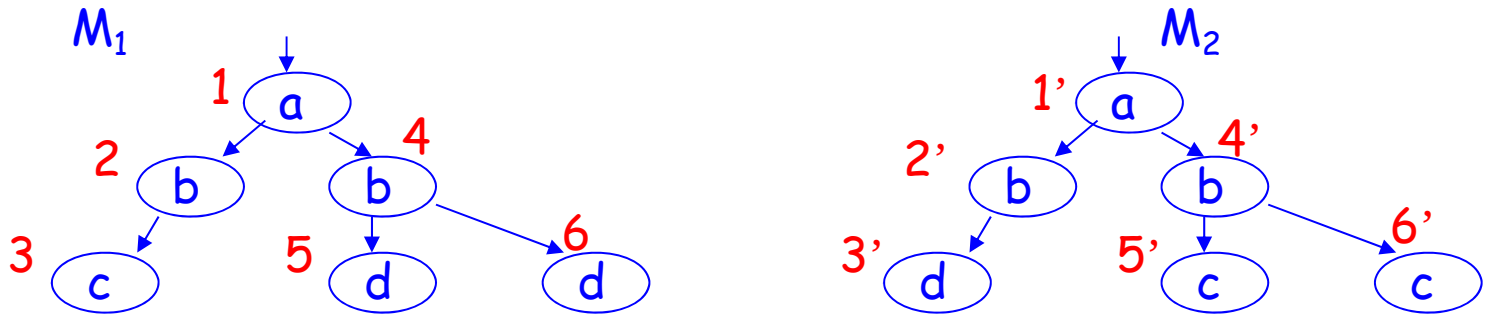
Bisimulation relation (cont.)

$B \subseteq S_1 \times S_2$ is a
Bisimulation between M_1 and M_2 iff

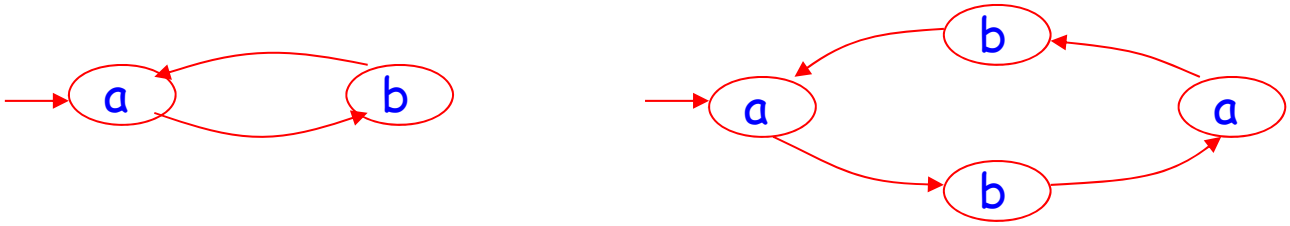
- B is a bisimulation, and
- for every $s_1 \in I_1$ there is $s_2 \in I_2$ s.t. $(s_1, s_2) \in B$ and
- for every $s_2 \in I_2$ there is $s_1 \in I_1$ s.t. $(s_1, s_2) \in B$

Notation: $M_1 \equiv M_2$

Bisimulation equivalence $M_1 \equiv M_2$

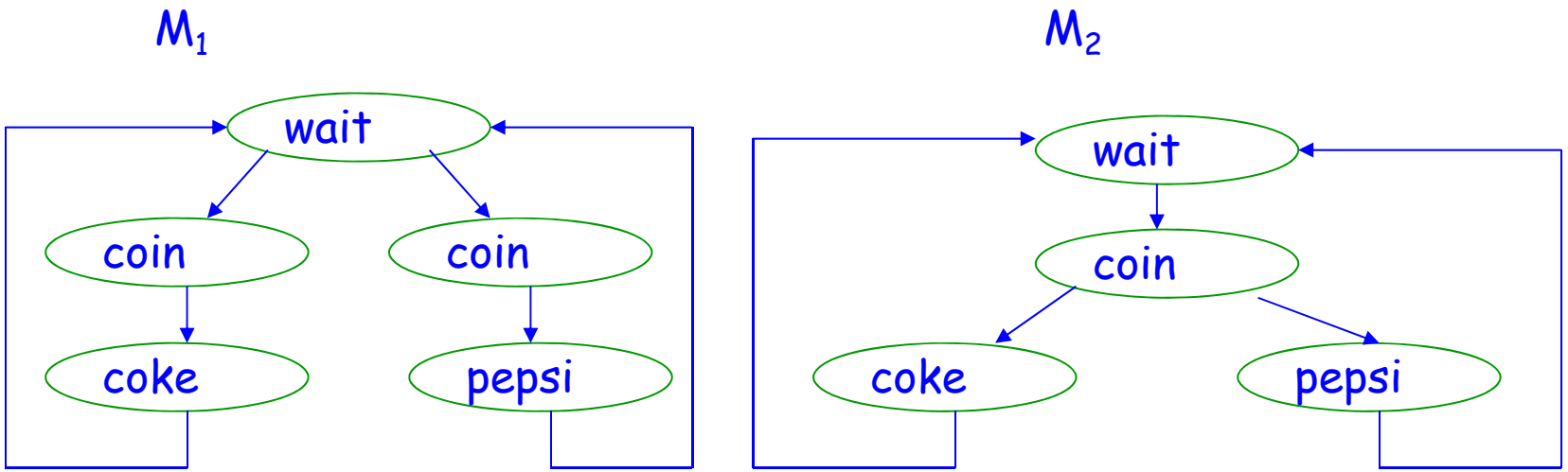


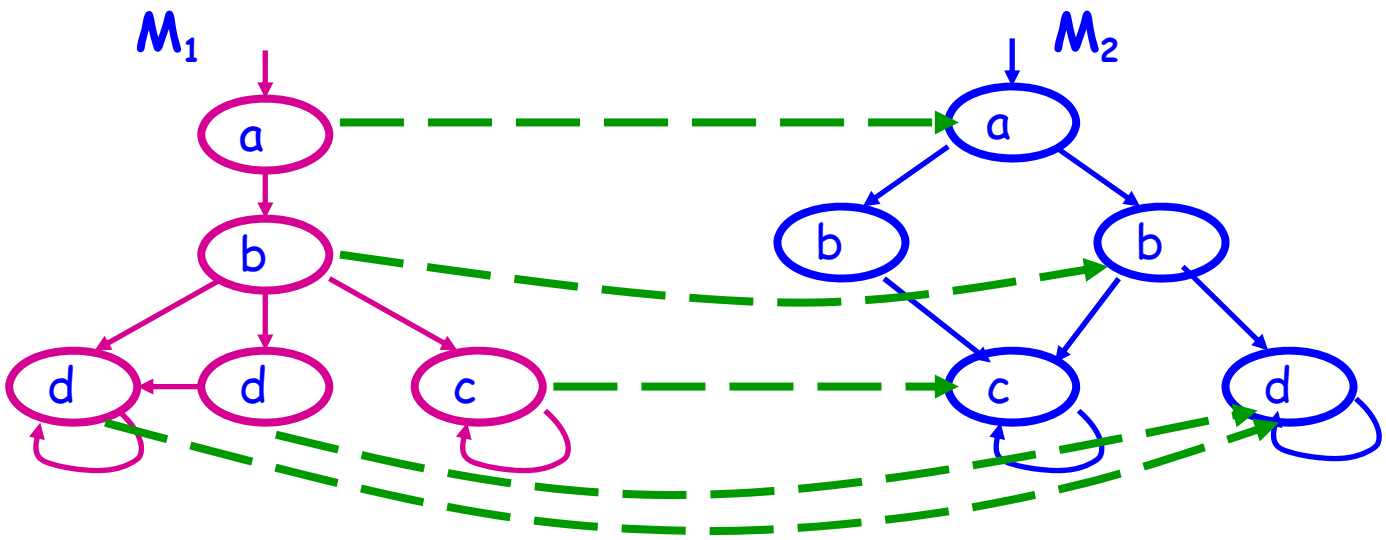
B={ (1,1'), (2,4'), (4,2'), (3,5'), (3,6'), (5,3'), (6,3') }



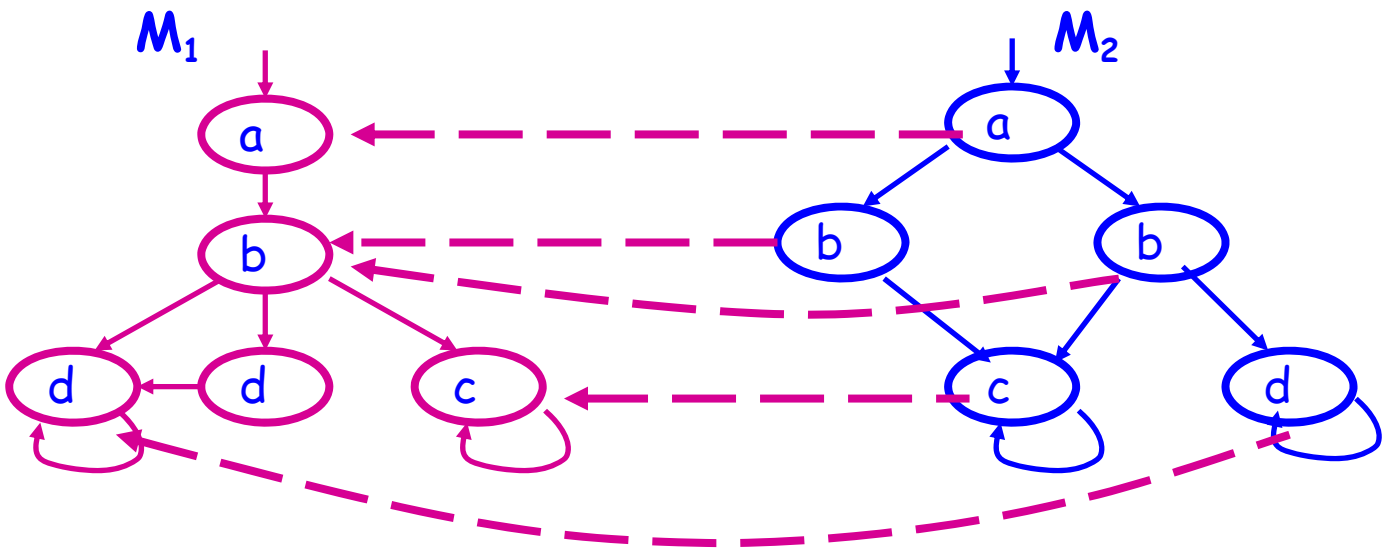
Simulation preorder

$M_1 \leq M_2$

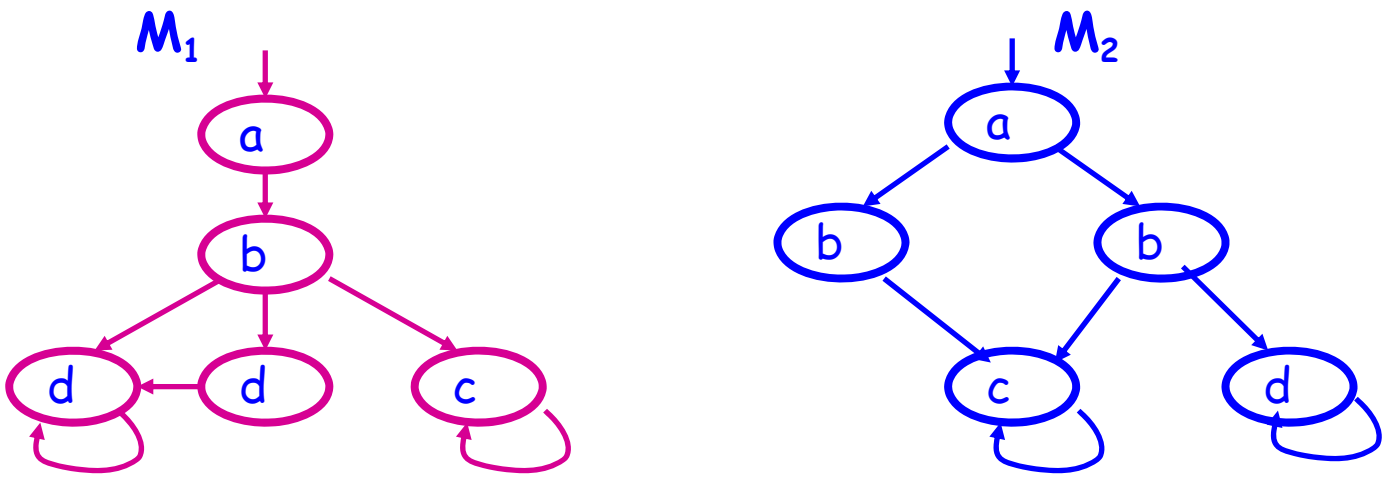




$M_1 \leq M_2$



$M_1 \geq M_2$



$M_1 \leq M_2$ and $M_1 \geq M_2$ but not $M_1 \equiv M_2$

since they do not agree on all CTL.

Example: $M_2 \models$ EX AX c $M_1 \not\models$ EX AX c

(bi)simulation and logic preservation

Theorem:

If $\mathbf{M_1} \equiv \mathbf{M_2}$ then for every **CTL*** formula φ,

$M_1 \models \varphi \Leftrightarrow M_2 \models \varphi$

If $\mathbf{M_2} \geq \mathbf{M_1}$ then for every **ACTL*** formula φ,

$M_2 \models \varphi \Rightarrow M_1 \models \varphi$

Lemma:

If $B(s,s')$ then

- for every path $\pi = s_0, s_1, \ldots$ from s there is a path $\pi' = s'_0, s'_1, \ldots$ from s' such that for every i: $B(s_i,s'_i)$
- for every path $\pi' = s'_0, s'_1, \ldots$ from s' there is a path $\pi = s_0, s_1, \ldots$ from s such that for every i: $B(s_i,s'_i)$

We say that π and π' correspond and write $B(\pi, \pi')$

Proof:

Assume $B(s,s')$ and let $\pi = s_0, s_1, \ldots$ be a path from s.

We construct $\pi' = s'_0, s'_1, \ldots$ from s' by induction on the location i on π'.

Base:

We choose s'_0 to be s'. Therefore $B(s_0,s'_0)$.

Inductive step:

Assume $B(s_i,s'_i)$. $R(s_i, s_{i+1})$ since they are consecutive on π.

Therefore, there is t' such that $R(s'_i, t')$ and $B(s_{i+1},t')$. We choose s'_{i+1} to be t'.

The proof that for every π' there is a corresponding π is similar

Proof (continued):

Note: induction can prove a property only for a finite (possibly unbounded) set.

Not for infinite sets.

Here: π is infinite.

We proved that for every prefix of π there is a corresponding prefix of π'

Proof (continued):

Assume there is no path starting from s' that corresponds to π.
Then for every path from s' there is an i such that $B(s_i,s'_i)$ does not hold.
But this contradicts the previous proof which shows that π' we constructed has $B(s_j, s'_j)$ for every j

Theorem:

Let B(**s**, **s'**). Then for every **CTL*** formula f,

$s \models f \Leftrightarrow s' \models f$

Proof:

We show a simpler proof for CTL.

By induction of the structure of the formula.

Base:

- $f \in AP$

Step:

- $f = \neg f_1$
- $f = f_1 \vee f_2$

- $f = EX\ f_1$
- $f = E\ (f_1\ U\ f_2)$
- $f = EG\ f_1$

Abstractions

- They are one of the most useful ways to **fight** the state explosion problem
- They should **preserve properties of interest**: properties that hold for the abstract model should hold for the concrete model
- Abstractions should be **constructed directly from the program**

Abstraction

- Removes or simplifies details
- Removes entire components

that are irrelevant to the property under consideration, thus reducing the model size (number of states and transitions)

- Manual abstraction requires great creativity

- Goal:
  Automatically construct an abstract model that will preserve the required property

Use

- In model checking, a small abstract model M_A will replace the full, concrete model M

- The abstract model M_A has
  - less states and transitions
  - More behaviors
- M_A is an over-approximation of M

- M_A preserves ACTL / ACTL* properties
  - If $M_A \models f$ then $M \models f$

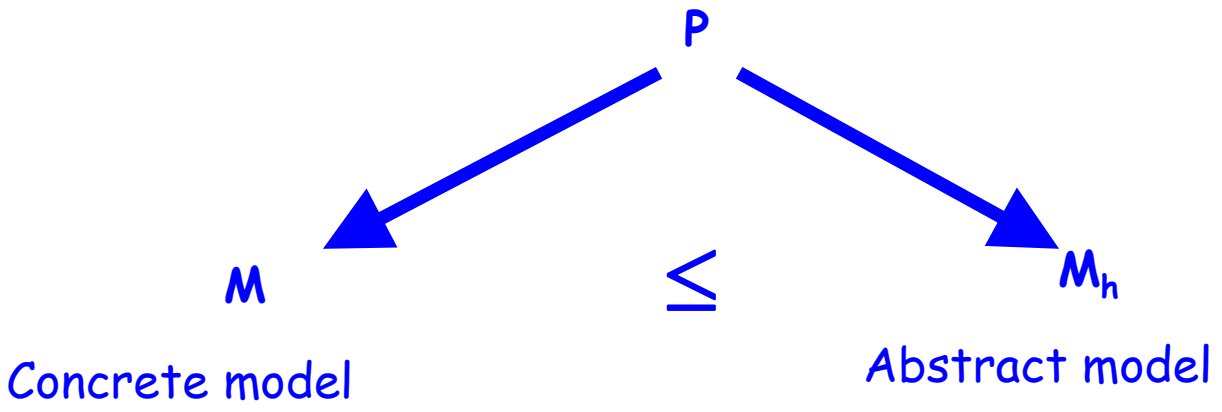
Outline for abstraction

- **Define** an abstract model that preserves the checked property

- Consider different **types** of abstractions
- **Automatically construct** an abstract model
  - Different constructions for different types

- **Automatically refine** it, if the abstraction is not detailed enough

- We first define an abstract model M_h based on a concrete (full) model M of the system
- Goal: constructing M_h directly from the program text



Abstraction preserving ACTL/ACTL*

We use **Existential Abstraction** in which the abstract model is an **over-approximation** of the concrete model:

- The abstract model has **more behaviors**
- But no concrete behavior is lost

- Every ACTL/ACTL* property true in the abstract model is also true in the concrete model

ACTL*

- ACTL* is a subset of CTL* where
  - only universal path quantifiers are used
  - negation is restricted to atomic formulas

AG AF restart is an ACTL* formula

Existential Abstraction

Given an abstraction function $h : S \rightarrow S_h$, the concrete states are grouped and mapped into abstract states :