Introduction to Software Verification

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Lectures Material
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Lecture 10

Symbolic (BDD-based) Model Checking for CTL

BDD-based Model Checking

- Accept: Kripke structre M, CTL formula f
- Returns: S_f - the set of states satisfying f

M is given by:

- BDD R(V,V'), representing the transition relation
- BDD p(V), for every $p \in AP$, representing S_p
  - the set of states satisfying p
- $V = (v_1,\ldots v_n)$

BDD-based Model Checking

- The algorithm works from simpler formulas to more complex ones
- When a formula g is handled, the BDD for S_g is built
- A formula is handled only after all its sub-formulas have been handled

BDD-based Model Checking

- For $p \in AP$, return $p(V)$
- For $f = f_1 \wedge f_2$, return $f(V) = f_1(V) \wedge f_2((V)$ (using apply)
- For $f = \neg f_1$, return $f(v) = \neg f_1(V)$

BDD-based Model Checking

- For $f = EX\ f_1$ return

  $f(V) = \exists V' [\ f_1(V') \wedge R(V,V')\ ]$

- This BDD represents all (encoding V of) states that have a successor (with encoding V') in f_1

- Defined as a new BDD operator:
  $EX\ f_1(V) = \exists V' [\ f_1(V') \wedge R(V,V')\ ]$

- This operation is also called pre-image

- Important:
  the formula defines a sequence of BDD operations and therefore is considered as a symbolic algorithm

Model Checking $f = E[g_1 \; U \; g_2]$

Given: BDDs $R(V,V')$, $g_1(V)$ and $g_2(V)$:

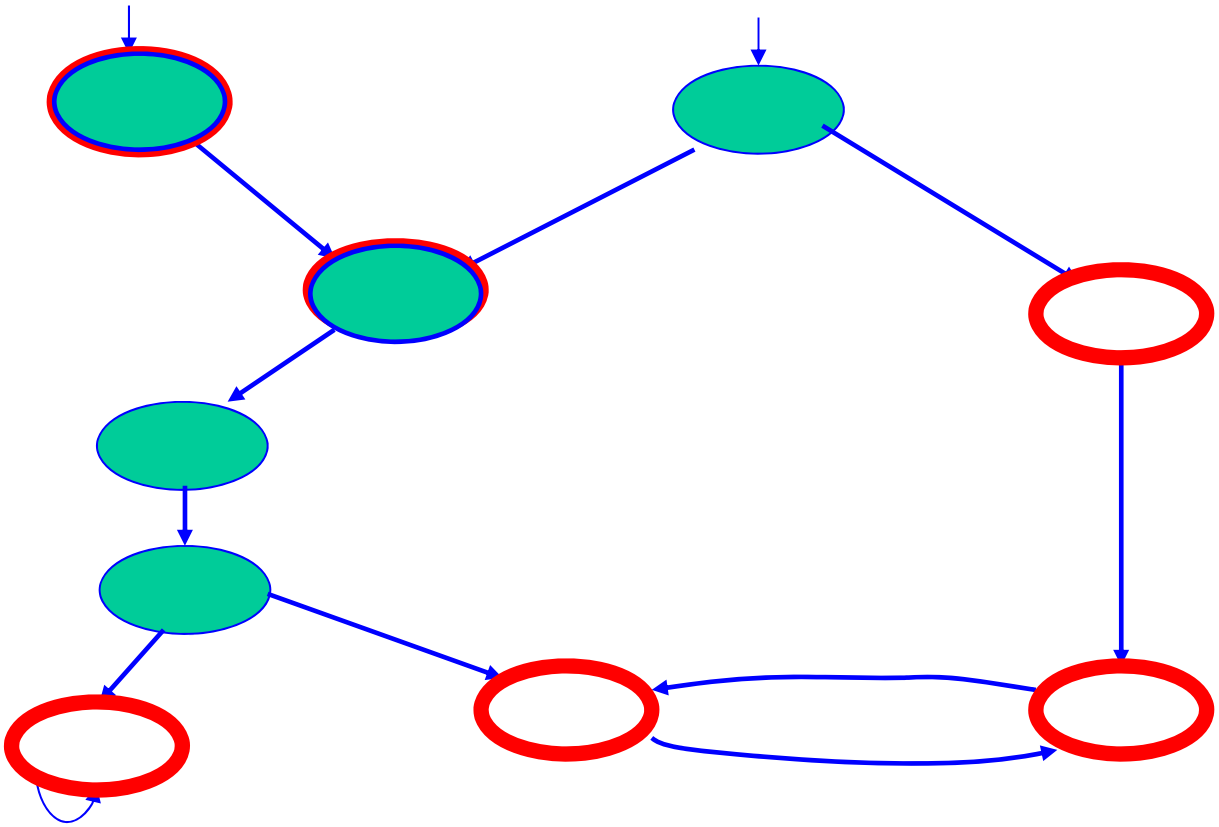
```
procedure CheckEU (g1, g2)
    Q := emptyset;  Q' := g2 ;
    while Q ≠ Q'  do
      Q := Q';
      Q' := Q ∨ (EX(Q) ∧ g1 )
    end while
    f := Q ;   return(f)
```

Model Checking f = EG g

Given: BDDs R(V,V') , g(V)

```
procedure CheckEG (g)
    Q := S ;  Q' := g ;
    while Q ≠ Q' do
      Q := Q';
      Q' := Q ∧ EX (Q)
    end while
    f := Q ;   return( f )
```

Example: f = EG g



Bounded (SAT-based) Model Checking

State explosion problem - revisited

- state of the art symbolic model checking can handle effectively designs with a few hundreds of Boolean variables

Other solutions for the state explosion problem are needed!

SAT-based model checking

- Translates the model and the specification to a propositional formula
- Uses efficient tools (SAT solvers) for solving the satisfiability problem

Since the satisfiability problem is **NP-complete**, SAT solvers are based on **heuristics**.

SAT-based model checking

Main idea

- Translate the model and the specification to propositional formulas
- Use efficient tools (SAT solvers) for solving the satisfiability problem

SAT tools

- Using heuristics, SAT tools can solve very large problems fast.
- They can handle systems with 1000 variables that create formulas with a few millions of variables.

GRASP (Silva, Sakallah)
Prover (Stalmark)
Chaff (Malik)
MiniSAT
Glucose

Bounded model checking (BMC) for checking AGp

- Given
  - A finite system M
  - A safety property AGp
  - A bound k

- Determine
  - Does M contain a counterexample to AGp of *k transitions (or fewer)* ?

Bounded Model Checking (BMC) for checking AGp

- Unwind the model for k levels, i.e., construct all computations of length k
- If a state satisfying $\neg p$ is encountered, produce a counterexample; Otherwise, increase k

[BCCZ 99]

Bounded Model Checking

Terminates

- with a counterexample or
- with time- or memory-out

The method is suitable for **falsification**, not verification

BMC for checking AGp (EF¬p)

Input to BMC:

A system over variables $V = \{v_1,\ldots,v_n\}$, where

- INIT(V) is a propositional formula representing the set of initial states
- R(V,V') is a propositional formula representing the transition relation

A specification:

- ¬p(V) is a propositional formula representing the set of states satisfying ¬p

BMC for checking $\varphi = EF\neg p$

1. k=1
2. Build a propositional formula f_M^k describing all prefixes of length k of paths of M from an initial state
3. Build a propositional formula f_φ^k describing all prefixes of length k of paths satisfying φ
4. If $(f_M^k \wedge f_\varphi^k)$ is **satisfiable**, return the satisfying assignment as a **counterexample**
5. Otherwise, increase k and return to 2.

- If $(f_M^k \wedge f_\varphi^k)$ is unsatisfiable:
  M has no counterexample of length k

- If $k = 2^{|V|}$ then we can conclude $M \models AGp$
  - Too big - not practical

- The method is suitable for refutation
  - Bug finding

BMC for checking $\varphi = EF\neg p$

- $f_M^k\ (V_0,\ldots,V_k) =$
  $INIT(V_0) \wedge R(V_0,V_1) \wedge \ldots \wedge R(V_{k-1},V_k)$

- Uses $k+1$ copies of $V = \{ v_1, \ldots, v_n \}$
- V_i represents the state after i transitions

BMC for checking $\varphi = EF\neg p$

- To check if p is violated within k steps:

  $f_\varphi^k(V_0,\ldots,V_k) =$
  $\neg p(V_0) \vee \ldots \vee \neg p(V_k) = V_{i=0\cdots k} \neg p(V_i)$

- To check if p is violated exactly on state k:
  $f_\varphi^k(V_0,\ldots,V_k) = \neg p(V_k)$
  - Useful when working iteratively on k=0,1,2,…

BMC for checking φ=EF¬p

- The iterative algorithm:

$$INIT(V_0) \wedge \neg p(V_0)$$

$$INIT(V_0) \wedge R(V_0,V_1) \wedge \neg p(V_1)$$

$$INIT(V_0) \wedge R(V_0,V_1) \wedge R(V_1,V_2) \wedge \neg p(V_2)$$

.

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$$INIT(V_0) \wedge R(V_0,V_1) \wedge R(V_1,V_2) \wedge \ldots \wedge R(V_{k-1},V_k) \wedge \neg p(V_k)$$

Example – shift register

Shift register of 3 bits: <x, y, z>

Transition relation:

$R(x,y,z,x',y',z') = \quad x'=y \ \wedge \ y'=z \ \wedge \ z'=1$

error

Initial condition:

$INIT(x,y,z) = \ x=0 \vee y=0 \vee z=0$

Specification: $AG\ (\ x=0 \vee y=0 \vee z=0)$

Propositional formula for k=2

$$f_{M,2} = (x_0=0 \vee y_0=0 \vee z_0=0) \wedge (x_1=y_0 \wedge y_1=z_0 \wedge z_1=1) \wedge (x_2=y_1 \wedge y_2=z_1 \wedge z_2=1)$$

INIT = $x=0 \vee y=0 \vee z=0$
R = $x'=y \wedge y'=z \wedge z'=1$

$$f_{\varphi,2} = V_{i=0,..2} (x_i=1 \wedge y_i=1 \wedge z_i=1)$$

p = $x=0 \vee y=0 \vee z=0$

Satisfying assignment: 101 011 111

This is a counterexample!

BMC for checking AFp (φ=EG$\neg$p)

- Is there an infinite path in M
  - From an initial state
  - all of its states satisfying $\neg$p
  - Over k+1 states ?
- Must be a lasso

BMC for checking AFp ($\varphi = EG\neg p$)

An infinite path in M, from an initial state, over k+1 states, all satisfying $\neg p$:

- $f_M^k\ (V_0,\ldots,V_k) =$
  $INIT(V_0) \wedge \Lambda_{i=0,\cdots k-1}\ R(V_i,V_{i+1}) \wedge \mathbf{V}_{i=0,\cdots k-1}\ (V_k=V_i)$

- $V_k=V_i$ means bitwise equality: $\Lambda_{j=0,\cdots n}\ (v_{kj} \leftrightarrow v_{ij})$

- $f_\varphi^k\ (V_0,\ldots,V_k) = \Lambda_{i=0,\cdots k}\ \neg p\ (V_i)$

A remark

In order to describe a computation of length k by a propositional formula we need k+1 copies of the state variables.

With BDDs we use only two copies of current and next states.

Bounded model checking

- Can handle all of **LTL** formulas
- Can be used for **verification** by choosing k which is large enough
  - Need bound on length of the shortest counterexample.
    - *diameter* bound. The diameter is the maximum length of the shortest path between any two states.
- Using such k is often **not practical** due to the size of the model
  - Computing worst case diameter is exponential. Obtaining better bounds is sometimes possible, but generally intractable.

SAT-based verification

- Induction
- Interpolation
- IC3